Calculus and Vectors: Motion Along a Curve

Ex. 1 (Noncalculator)
A particle moves in the xy-plane so that at any time t, the position of the particle is given by $x(t) = t^3 + 4t^2$, $y(t) = t^4 - t^3$.

(a) Find the velocity vector when $t = 1$.

$x'(t) = 3t^2 + 8t$ $\quad y'(t) = 4t^3 - 3t^2$

$x'(1) = 11$ $\quad y'(1) = 1$

$v(t) = \langle 11, 1 \rangle$

(b) Find the acceleration vector when $t = 2$.

$x''(t) = 6t + 8$ $\quad y''(t) = 12t^2 - 6t$

$x''(2) = 20$ $\quad y''(2) = 36$

$a(t) = \langle 20, 36 \rangle$

Ex.2 (Noncalculator)
A particle moves in the xy-plane so that at any time t, $t \geq 0$, the position of the particle is given by $x(t) = t^2 + 3t$, $y(t) = t^3 - 3t^2$. Find the magnitude of the velocity vector when $t = 1$.

speed

$x'(t) = 2t + 3$ $\quad y'(t) = 3t^2 - 6t$

$x'(1) = 5$ $\quad y'(1) = -3$

$\text{speed} = \sqrt{(5)^2 + (-3)^2}$

$\text{speed} = \sqrt{34}$

Ex. 3 (Noncalculator)
A particle moves in the xy-plane so that $x = \sqrt{3} - 4\cos t$ and $y = 1 - 2\sin t$, where $0 \leq t \leq 2\pi$. The path of the particle intersects the x-axis twice. Write an expression that represents the distance traveled by the particle between the two x-intercepts. Do not evaluate.

$y = 0$

$1 - 2\sin t = 0$

$-2\sin t = -1$

$\sin t = 1/2$

$t = \frac{\pi}{6}, \frac{5\pi}{6}$

$x'(t) = 4\sin t$ $\quad y'(t) = -2\cos t$

$$d = \int_{\pi/6}^{5\pi/6} \sqrt{(4\sin t)^2 + (-2\cos t)^2}\, dt$$

Ex. 4 A particle moves in the xy-plane so that at any time t, the position of the particle is given by $x(t) = 2t^3 - 15t^2 + 36t + 5$, $y(t) = t^3 - 3t^2 + 1$, where $t \geq 0$. For what value(s) of t is the particle at rest?

$x'(t) = 0$ and $y'(t) = 0$

$x'(t) = 6t^2 - 30t + 36$

$6t^2 - 30t + 36 = 0$

$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0 \rightarrow t = 3, t = 2$

$y'(t) = 3t^2 - 6t$

$3t(t-2) = 0$

$t = 0, \; t = 2$

The particle is at rest when $t = 2$.

Ex. 5 A particle moves in the xy-plane in such a way that its velocity vector is $\langle 3t^2 - 4t, 8t^3 + 5 \rangle$. If the position vector at $t = 0$ is $\langle 7, -4 \rangle$, find the position of the particle at $t = 1$.

$x'(t) = 3t^2 - 4t$

$x(t) = t^3 - 2t^2 + C$

$x(0) = 0^3 - 2(0)^2 + C$

$7 = C$

$x(t) = t^3 - 2t^2 + 7$

$x(1) = (1)^3 - 2(1)^2 + 7$

$x(1) = 6$

$y'(t) = 8t^3 + 5$

$y(t) = 2t^4 + 5t + C$

$y(0) = 2(0)^4 + 5(0) + C$

$-4 = C$

$y(t) = 2t^4 + 5t - 4$

$y(1) = 2(1)^4 + 5(1) - 4$

$y(1) = 3$

$x(1) = \langle 6, 3 \rangle$