CALCULUS BC
WORKSHEET 1 ON VECTORS

Work the following on **notebook paper.** Use your calculator on problems 10 and 13c only.

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\dfrac{dy}{dx}$.

2. If a particle moves in the *xy*-plane so that at any time $t > 0$, its position vector is $\left\langle \ln\left(t^2 + 5t\right),\ 3t^2 \right\rangle$, find its velocity vector at time $t = 2$.

3. A particle moves in the *xy*-plane so that at any time *t*, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

4. If a particle moves in the *xy*-plane so that at time *t* its position vector is $\left\langle \sin\left(3t - \dfrac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \dfrac{\pi}{2}$.

5. A particle moves on the curve $y = \ln x$ so that its *x*-component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point (1, 0). Find the position of the particle at time $t = 1$.

6. A particle moves in the *xy*-plane in such a way that its velocity vector is $\left\langle 1 + t,\ t^3 \right\rangle$. If the position vector at $t = 0$ is $\left\langle 5,\ 0 \right\rangle$, find the position of the particle at $t = 2$.

7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\dfrac{dy}{dt} = 3$, what is the value of $\dfrac{dx}{dt}$?

8. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - \dfrac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of *t* is the particle at rest?

9. A curve *C* is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of *C* at the point $(8,\ -4)$.

10. A particle moves in the *xy*-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$ Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \dfrac{2}{3}t^3$.

    (a) Find the magnitude of the velocity vector at time $t = 5$.

    (b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$. (c) Find $\dfrac{dy}{dx}$ as a function of *x*.

12. Point $P(x,\ y)$ moves in the *xy*-plane in such a way that $\dfrac{dx}{dt} = \dfrac{1}{t+1}$ and $\dfrac{dy}{dt} = 2t$ for $t \geq 0$.

    (a) Find the coordinates of *P* in terms of *t* given that $t = 1$, $x = \ln 2$, and $y = 0$.
    (b) Write an equation expressing *y* in terms of *x*.
    (c) Find the average rate of change of *y* with respect to *x* as *t* varies from 0 to 4.
    (d) Find the instantaneous rate of change of *y* with respect to *x* when $t = 1$.

13. Consider the curve *C* given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$.

    (a) Find $\dfrac{dy}{dx}$ as a function of *t*. (b) Find the equation of the tangent line at the point where $t = \dfrac{\pi}{4}$.

    (c) The curve *C* intersects the *y*-axis twice. Approximate the length of the curve between the two *y*-intercepts.

Answers to Worksheet 1 on Vectors

1. $\dfrac{dy}{dx} = \dfrac{3t^2 e^{t^3}}{2t} = \dfrac{3te^{t^3}}{2}$

2. $\left\langle \dfrac{9}{14}, 12 \right\rangle$

3. $\langle 20, 24 \rangle$

4. $\langle -3, 3\pi \rangle$

5. $\left(\dfrac{5}{2}, \ln\left(\dfrac{5}{2} \right) \right)$

6. $(9, 4)$

7. $-\dfrac{6}{5}$

8. $t = 3$

9. $y + 4 = -\dfrac{1}{12}(x - 8)$

10. $\langle 7.008, -2.228 \rangle$

11. (a) $\sqrt{2600}$ or $10\sqrt{26}$ (b) $\dfrac{2}{3}\left(26^{3/2} - 1 \right)$ (c) $t = \sqrt{x+3}$

12. (a) $\left(\ln(t+1), t^2 - 1 \right)$ (b) $y = \left(e^x - 1 \right)^2 - 1$ or $y = e^{2x} - 2e^x$.

(c) $\dfrac{16}{\ln 5}$ (d) 4

13. (a) $\dfrac{2}{3}\cot t$ (b) $y - \left(3 + \sqrt{2} \right) = \dfrac{2}{3}\left(x - \left(2 - \dfrac{3\sqrt{2}}{2} \right) \right)$ (c) 3.756