Vectors Day 2

1) $x = e^{2t}$ $y = \sin(3t)$

$\frac{dx}{dt} = 2e^{2t}$ $\frac{dy}{dt} = 3\cos 3t$

$$\frac{dy}{dx} = \frac{3\cos 3t}{2e^{2t}}$$

2) $x = (\cos t)^3$ $y = (\sin t)^2$

$\frac{dx}{dt} = 3\cos^2 t(-\sin t)$ $\frac{dy}{dt} = 2\sin t\cos t$

$$S = \int_0^{\pi/2} \sqrt{(-3\sin t\cos^2 t)^2 + (2\sin t\cos t)^2}$$

3) $v(t) = \langle 3t^2 - 2t, 4t^3 + 4t - 8 \rangle$

$3t^2 - 2t = 0$

$t(3t - 2) = 0$

$t = 0 \quad t = 2/3$

4) $x = t^2 + 1$ $y = \ln(2t + 3)$

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = \frac{2}{2t+3} = 2(2t+3)^{-1}$

$\frac{d^2x}{dt^2} = 2$ $\frac{d^2y}{dt^2} = \frac{-2}{(2t+3)^2}(2) = \frac{-4}{(2t+3)^2}$

$$a(t) = \left\langle 2, \frac{-4}{(2t+3)^2} \right\rangle$$

5) $v(t) = \langle 6t - 4, 3t^2 - 4 \rangle$

$\frac{dy}{dx} = \frac{3t^2 - 4}{6t - 4} = \frac{3-4}{6-4} = \frac{-1}{2}$

$x(1) = 3 - 4 + 2 = 1$

$y(1) = 1 - 4 = -3$

$$y + 3 = \frac{-1}{2}(x - 1)$$

6) $x = e^t + 1$ $y = 2e^{2t}$

$x - 1 = e^t$ $y = 2e^{2\ln|x-1|}$

$t = \ln|x - 1|$ $y = 2e^{\ln|x-1|^2}$

$y = 2(x-1)^2$

$$y = 2x^2 - 4x + 2$$

7) $x = \cos(5t)$ $y = t^3$

$v(t) = \langle -5\sin(5t), 3t^2 \rangle$

$v(2) = \langle -5\sin(10), 12 \rangle$

speed $= \sqrt{(-5\sin(10))^2 + (12)^2}$

speed $= 12.304$

8) $x(t) = \frac{(t-2)^3}{3} + 4$, $y(t) = t^2 - 4t + 4$

$\frac{dx}{dt} = (t-2)^2$ $\frac{dy}{dt} = 2t - 4$

speed $= \sqrt{(-1)^2 + (-2)^2}$

a) speed $= \sqrt{5}$

$$S = \int_0^1 \sqrt{((t-2)^2)^2 + (2t-4)^2}\, dt$$

b) $S = 3.816$

c) $(t-2)^2 = 0$ $2t - 4 = 0$

$t = 2$

c) at rest $t = 2$

$x(2) = 4$ $y(2) = 0$

position $(4, 0)$

9) $\frac{dx}{dt} = 1 + \tan(t^2)$ $\frac{dy}{dt} = 3e^{\sqrt{t}}$

$a(t) = \left\langle 2t\sec^2(t^2), \frac{3}{2\sqrt{t}} e^{\sqrt{t}} \right\rangle$

$a(5) = \left\langle 2(5)\sec^2(25), \frac{3}{2\sqrt{5}} e^{\sqrt{5}} \right\rangle$

$$a(5) = \langle 10.178, 6.277 \rangle$$

speed $= \sqrt{(1 + \tan 25)^2 + (3e^{\sqrt{5}})^2}$

speed $= 28.083$

10) $x(t) = t + \cos t$

$y(t) = 3t + 2\sin t$

$v(t) = \langle 1 - \sin t, 3 + 2\cos t \rangle$

$5 = 3t + 2\sin t$

$t = 1.079$

$v(1.079) = \langle 1 - \sin(1.079), 3 + 2\cos(1.079) \rangle$

$$v(1.079) = \langle .119, 3.944 \rangle$$

11) $\frac{dx}{dt} = 2\sin(t^3)$ $\frac{dy}{dt} = \cos(t^2)$ $t=1$ $x(t)=(3,4)$

a) $\frac{dy}{dx} = \frac{\cos(t^2)}{2\sin(t^3)} = \frac{\cos(1)}{2\sin(1)} = .321$

$y-4 = .321(x-3)$

b) $t=2$ speed $= \sqrt{(\cos 4)^2 + (2\sin 8)^2}$

speed $= 2.084$

c) $S = \int_0^1 \sqrt{(\cos(t^2))^2 + (2\sin(t^3))^2}\, dt$

$S = 1.126$

$\frac{dx}{dt} = 2\sin(t^3)$ $\frac{dy}{dt} = \cos(t^2)$

$\int_1^2 x'(t)\,dt = x(2) - x(1)$ $\int_1^2 y'(t)\,dt = y(2) - y(1)$

$-x(2) = -x(1) - \int_1^2 2\sin(t^3)\,dt$ $-y(2) = -y(1) - \int_1^2 y'(t)\,dt$

$-x(2) = -3 - \int_1^2 2\sin(t^3)\,dt$ $-y(2) = -4 - \int_1^2 \cos(t^2)\,dt$

$-x(2) = -3.436$ $-y(2) = -3.557$

$x(2) = 3.436$ $y(2) = 3.557$

d) position at $t=2$ is $\langle 3.436, 3.557 \rangle$