Calculus Vectors: Motion Along a Curve Day 2

Ex. (Calculator)

An object moving along a curve in the *xy*-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = \sin(t^3)$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at the position (1, 4).

(a) Find the acceleration vector for the particle at $t = 2$.

$x''(t) = \cos(t^3)\,3t^2$ $\quad y''(t) = -\sin(t^2)\,2t$

$x''(2) = 12\cos(8)$ $\quad y''(2) = -4\sin(4)$

$\boxed{a(2) = \langle -1.746, 3.027 \rangle}$

(b) Write the equation of the tangent line to the curve at the point where $t = 2$.

$\frac{dy}{dx} = \frac{\cos(4)}{\sin(8)}$

$\frac{dy}{dx} = -.661$

$\boxed{y - 4 = -.661(x - 1)}$

(c) Find the speed of the vector at $t = 2$.

$\text{Speed} = \sqrt{(\sin 8)^2 + (\cos 4)^2}$

$\boxed{\text{Speed} = 1.186}$

(d) Find the position of the particle at time $t = 1$.

$x'(t) = \sin(t^3)$

$\int_1^2 x'(t) = x(2) - x(1)$

$\int_1^2 \sin(t^3)\,dt = 1 - x(1)$

$x(1) = 1 - \int_1^2 \sin(t^3)\,dt$

$x(1) = .782$

$y'(t) = \cos(t^2)$

$\int_1^2 y'(t)\,dt = y(2) - y(1)$

$\int_1^2 \cos(t^2)\,dt = 4 - y(1)$

$y(1) = 4 - \int_1^2 \cos(t^2)\,dt$

$y(1) = 4.443$

$\boxed{x(1) = \langle .782, 4.443 \rangle}$