CALCULUS BC
WORKSHEET 2 ON VECTORS

Work the following on **notebook paper**. Use your calculator on problems 7 – 11 only.

1. If $x = e^{2t}$ and $y = \sin(3t)$, find $\frac{dy}{dx}$ in terms of t.

2. Write an integral expression to represent the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^2 t$ for $0 \le t \le \frac{\pi}{2}$.

3. For what value(s) of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

4. For any time $t \ge 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t + 3)$, find the acceleration vector.

5. Find the equation of the tangent line to the curve given by the parametric equations $x(t) = 3t^2 - 4t + 2$ and $y(t) = t^3 - 4t$ at the point on the curve where $t = 1$.

6. If $x(t) = e^t + 1$ and $y = 2e^{2t}$ are the equations of the path of a particle moving in the xy-plane, write an equation for the path of the particle in terms of x and y.

7. A particle moves in the xy-plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?

8. The position of a particle at time $t \ge 0$ is given by the parametric equations $x(t) = \frac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.
   (a) Find the magnitude of the velocity vector at $t = 1$.
   (b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.
   (c) When is the particle at rest? What is its position at that time?

9. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time with $\frac{dx}{dt} = 1 + \tan(t^2)$ and $\frac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.

10. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = t + \cos t$ and $y(t) = 3t + 2\sin t$, $0 \le t \le \pi$. Find the velocity vector when the particle's vertical position is $y = 5$.

11. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 2\sin(t^3)$ and $\frac{dy}{dt} = \cos(t^2)$ for $0 \le t \le 4$. At time $t = 1$, the object is at the position (3, 4).
   (a) Write an equation for the line tangent to the curve at (3, 4).
   (b) Find the speed of the object at time $t = 2$.
   (c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
   (d) Find the position of the object at time $t = 2$.

12. A particle moving along a curve in the xy-plane has position $\left(x(t),\ y(t)\right)$ at time t with $\frac{dx}{dt} = \arcsin\left(\frac{t}{t+4}\right)$ and $\frac{dy}{dt} = \ln\left(t^2+3\right)$. At time $t = 1$, the particle is at the position (5, 6).

(a) Find the speed of the object at time $t = 2$.

(b) Find the total distance traveled by the object over the time interval $1 \le t \le 2$.

(c) Find $y(2)$.

(d) For $0 \le t \le 3$, there is a point on the curve where the line tangent to the curve has slope 8. At what time t, $0 \le t \le 3$, is the particle at this point? Find the acceleration vector at this point.

Answers to Worksheet 2 on Vectors

1. $\dfrac{3\cos(3t)}{2e^{2t}}$

2. $\displaystyle\int_0^{\pi/2} \sqrt{9\cos^4 t\sin^2 t + 4\sin^2 t\cos^2 t}\, dt$

3. $t = 0$ and $t = \dfrac{2}{3}$

4. $v(t) = \left\langle 2t,\ \dfrac{2}{2t+3} \right\rangle$, $a(t) = \left\langle 2,\ -\dfrac{4}{(2t+3)^2} \right\rangle$

5. $y + 3 = -\dfrac{1}{2}(x-1)$

6. $y = 2x^2 - 4x + 2.$

7. 12.304

8. (a) $\sqrt{5}$ (b) 3.816 (c) At rest when t = 2. Position = (4, 0)

9. $a(5) = \langle 10.178,\ 6.277 \rangle$, speed = 28.083

10. $\langle 0.119,\ 3.944 \rangle$

11. (a) $y - 4 = 0.321(x-3)$ (b) 2.084 (c) 1.126 (d) (3.436, 3.557)

12. (a) 2.061 (b) 1.738 (c) 7.661 (d) $\langle 0.422,\ 0.179 \rangle$