Calculus Section 2.1 and 2.2 Tangent Lines

-Write the equation of a tangent line for a function at a point.
-Find horizontal tangent lines from the derivative.
-Graphs and derivatives

Homework: Page 103 #'s 25-33 odd, 34, 37, 39, 42
Page 115 #'s 57-61 odd, 80

Finding the equation of the tangent line of a function at a point.
To find the equation of a tangent line at a point, follow these steps:

1) find the equation for slope by taking the derivative
2) substitute x into the derivative to find the slope
3) write an equation using the slope in point-slope form

Examples)

1) $y = x^4 - 3x^2 + 2$ at (1,0)

$y' = 4x^3 - 6x$

$y' = 4(1)^3 - 6(1)$

$y' = -2$

$y - 0 = -2(x-1)$

2) $y = x^3 + x$ at (-1,-2)

$y' = 3x^2 + 1$

$y' = 3(-1)^2 + 1$

$y' = 4$

$y + 2 = 4(x+1)$

Finding a horizontal tangent line
To find where a function has a horizontal tangent line:

1) take the derivative of the function
2) set the derivative equal to zero.

Examples) Find where the graph has a horizontal tangent line (if any exist)

1) $y = \frac{1}{3}x^3 - 4x$

$y' = x^2 - 4$

$y' = (x+2)(x-2)$

$0 = (x+2)(x-2)$

$x = -2 \quad x = 2$

2) $y = \sin x + 5$

$y' = \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

3) $y = 6x - 4$

$y' = 6$

No horizontal tangents

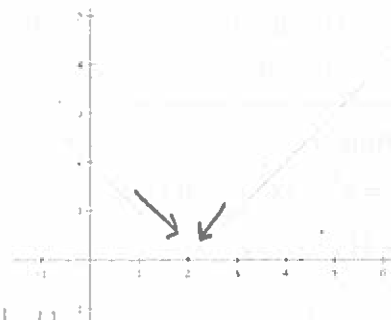
Differentiability and Continuity

There is an alternate definition of a derivative using limits that is useful when investigating the relationship between differentiability and continuity. The derivative of *f* at *c* is $f'(c) = \dfrac{f(x)-f(c)}{x-c}$. The existence of this limit requires that the one-sided limits exist and are equal: formula for slope

$$\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c} = \lim_{x\to c^+}\frac{f(x)-f(c)}{x-c} \quad \text{or} \quad \lim_{x\to c^-} f'(x) = \lim_{x\to c^+} f'(x)$$

A Graph with a Sharp Turn

Consider the function f(x) = |x-2|



$$\lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2} = -1$$

and

$$\lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2} = 1$$

Since the one-sided limits are not equal, we can conclude ~~the~~ f(x) is not differentiable and has no tangent line at x=2. This is even though f(x) = |x-2| is everywhere continuous.

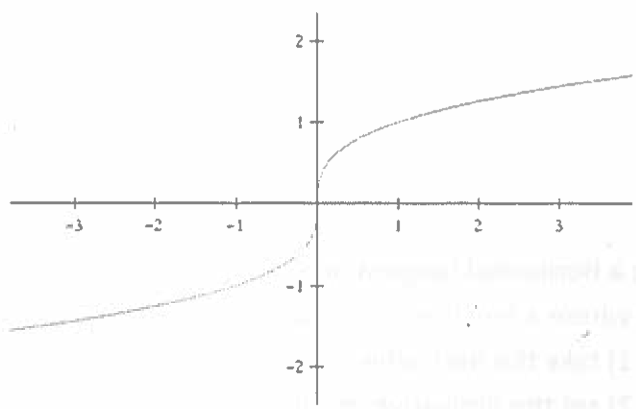
A Graph with a Vertical Tangent Line

Let $f(x) = x^{\frac{1}{3}}$. f(x) is continuous at x = 0 as shown in the drawing, but because the limit

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \frac{x^{1/3}-0}{x-0}$$

$$\lim_{x\to 0}\frac{x^{1/3}}{x}$$

$$\lim_{x\to 0}\frac{1}{x^{2/3}} = \frac{1}{0} \quad \text{undefined}$$



Differentiability Implies Continuity

If *f* is differentiable at x = c, then *f* is continuous at x = c.

This does not mean that if a function is continuous it is also differentiable. Graphs with either a sharp turn or a vertical tangent are not differentiable at the point where either of those actions occur.