Calculus Section 2.1 Definition of a Derivative

-Find the slope of the tangent line to a curve at a point.
-Use the limit definition to find the derivative of a function.

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Definition of the Derivative of a Function

The derivative of a function allows you to find the slope of a function at a point.

The derivative of *f* is given by:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Provided the limit exists. For all x for which this limit exists, f ' is a function of x.

Different ways to write the derivative:

1) $f'(x)$ 2) $\frac{dy}{dx}$ 3) y' 4) $\frac{d}{dx}[f(x)]$ 5) $D_x[y]$

Finding the derivative of a function using limits:

Find the derivative of f(x) = 2x – 3 (The answer should be 2 because it is the slope of y = mx + b)

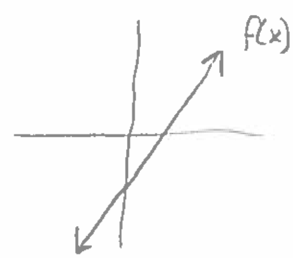
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{2(x+h) - 3 - (2x-3)}{h}$$

$$\lim_{h\to 0} \frac{2x + 2h - 3 - 2x + 3}{h}$$

$$\lim_{h\to 0} \frac{2h}{h} = 2$$

$$f'(x) = 2$$



The slope of f(x) = 2x-3 is 2 and is always 2 because the slope of a linear function doesn't change.

Find the derivative of f(x) = $x^3 + 2x$. Find the equation of the tangent line through the point (2, 12).

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{(x+h)^3 + 2(x+h) - (x^3+2x)}{h}$$

$$\lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h - x^3 + 2x}{h}$$

$$\lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 2h}{h} = 3x^2 + 3xh + h^2 + 2 = 3x^2 + 2$$

$f'(x) = 3x^2 + 2 \leftarrow$ The slope of the function is a function. The value of the slope (derivative) changes depending on the value of x.

$f'(2) = 3(2)^2 + 2$

$f'(2) = 14$

The slope of f(x) when x = 2 is 14.

$y - y_1 = m(x - x_1)$

$y - 12 = 14(x - 2)$

Alternate Definition of a Derivative

An alternative definition of a derivative is written:

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

This definition is based on the formula for slope. The derivative is the calculated slope between two points that are infinitesimally close together.

The existence of this alternative form requires that the one-sided limits

$$\lim_{x\to c^+} \frac{f(x) - f(c)}{x - c} \quad \text{and} \quad \lim_{x\to c^-} \frac{f(x) - f(c)}{x - c}$$

exist and are equal. Otherwise, the function has no derivative at point c.