Calculus Section 2.2 Rates of Change

-Find the average velocity on an interval
-Find the instantaneous velocity at a point

Homework: page 115 #'s 47-51 odd, 61, 70, 87-92, 97, 98, 101-105 odd

The function *s* that gives the position of an object as a function of time *t* is called a **position function**. If the object changes position over a period of time, Δt, the **average velocity** is given by the formula:

average velocity = $\frac{\Delta s}{\Delta t}$ $\frac{\text{change in position}}{\text{change in time}}$

Example) A tennis ball is dropped from a height of 100 feet, its height *s* at time *t* is given by the position function

$$s = -16t^2 + 100$$

where *s* is measured in feet and *t* is measured in seconds. Find the average velocity over each time interval.

1) [1,2]

Avg Vel = $\frac{s(2) - s(1)}{2 - 1}$

$= \frac{36 - 84}{1}$

$= -48$ ft/s

2) [1, 1.5]

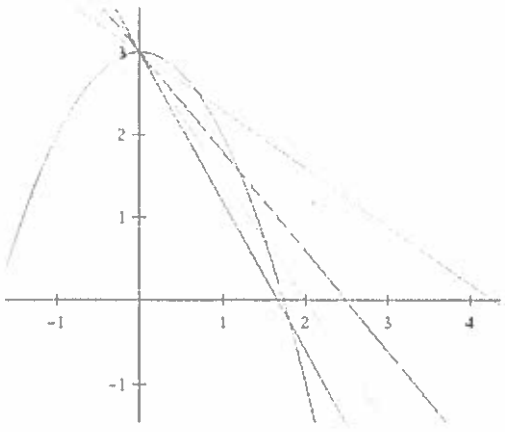
Avg Vel = $\frac{s(1.5) - s(1)}{1.5 - 1}$

$= \frac{64 - 84}{.5}$

$= 40$ ft/s

The average velocities are negative. This indicates that the tennis ball is moving downward.

Position equation vs. Velocity



As we decrease the interval used to find average velocity, we can find the **instantaneous velocity** of a function at a point.
So,

$$\lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = s'(t) = v(t)$$

In other words, the velocity is the derivative of the position function.
Velocity is a **vector**, which means it has direction. So, velocity can be positive, negative, or zero. **Speed** is the absolute value of velocity, and cannot be negative.

Example)

At time t=0, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t is measured in seconds.

a) When does the diver hit the water?
b) What is the diver's velocity at impact? What is their speed?
c) When does the diver stop moving upward and start their descent?

a) $0 = -16t^2 + 16t + 32$

$t = 2$

b) $v(t) = -32t + 16$

$v(2) = -48$ ft/s

c) $v(t) = 0$

$-32t + 16 = 0$

$t = 1/2$ s