2.3 Higher Order Derivatives and Product Rule

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1) $g(x) = (x^2+3)(x^2-4x)$

$g'(x) = (x^2+3)(2x-4) + (x^2-4x)(2x)$

$g'(x) = 2x^3 - 4x^2 + 6x - 12 + 2x^3 - 8x^2$

$\boxed{g'(x) = 4x^3 - 12x^2 + 6x - 12}$

3) $h(t) = \sqrt{t}(1-t^2) = t^{1/2}(1-t^2)$

$h'(t) = t^{1/2}(-2t) + (1-t^2)\left(\frac{1}{2}t^{-1/2}\right)$

$h'(t) = -2t^{3/2} + \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{3/2}$

$\boxed{h'(t) = -\frac{5}{2}t^{3/2} + \frac{1}{2\sqrt{t}}}$

5) $f(x) = x^3\cos x$

$f'(x) = x^3(-\sin x) + \cos x(3x^2)$

$\boxed{f'(x) = -x^3\sin x + 3x^2\cos x}$

17) $f(x) = x\cos x$

$f'(x) = x(-\sin x) + \cos x(1)$

$f'(x) = -x\sin x + \cos x$

$f'\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)$

$\boxed{f'\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}}$

62) $f(x) = \sin x(\sin x + \cos x)$

$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$

$f'\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(0) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{f'\left(\frac{\pi}{4}\right) = 1}$

63) $f(x) = (x^3+4x-1)(x-2)$

$f'(x) = (x^3+4x-1)(1) + (x-2)(3x^2+4)$

$f'(1) = (4)(1) + (-1)(7)$

$f'(1) = -3$

$\boxed{y+4 = -3(x-1)}$

81a) $p(x) = f(x)g(x)$

$p'(x) = f(x)g'(x) + g(x)f'(x)$

$p'(1) = f(1)g'(1) + g(1)f'(1)$

$p'(1) = (6)(-1/2) + (4)(1)$

$\boxed{p'(1) = 1}$

82a) $p'(4) = f(4)g'(4) + g(4)f'(4)$

$p'(4) = (8)(1/2) + (1)(0)$

$\boxed{p'(4) = 4}$

91) $f(x) = x^4 + 2x^3 - 3x^2 - x$

$f'(x) = 4x^3 + 6x^2 - 6x - 1$

$\boxed{f''(x) = 12x^2 + 12x - 6}$

93) $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2}$

$\boxed{f''(x) = \frac{3}{\sqrt{x}}}$

97) $f(x) = x \sin x$

$f'(x) = x\cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$\boxed{f''(x) = -x\sin x + 2\cos x}$

115) $v(t) = 36 - t^2$

$a(t) = v'(t) = -2t$

$a(t) = -2t$

$a(3) = -6\ m/s^2 \quad v(3) = 27\ m/s$

The speed of the object is slowing down because velocity and acceleration are different signs.

132) $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f'''(x) = 24x$

$f^{(4)}(x) = 24$

$f^{(5)}(x) = 0$

$\boxed{\text{True}}$

133) True

134) True