Calculus Section 2.3 Higher Order Derivatives & Product Rule

Homework: Page 125 #'s 1, 3, 5, 17, 62, 63, 81a, 82a, 91, 93, 97, 115, 132-134.

-Find higher order derivatives of functions.
-Find an equation for acceleration from a position function.
-Find the derivative of a function using the product rule.

Higher Order Derivatives

Oftentimes, more than one derivative can be taken for a differentiable function. These derivatives imply continued continuity (like the first derivative), and can be used to find helpful information about a function. Higher order derivatives can be denoted as follows:

First derivative:	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$
Second derivative:	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$
Third derivative:	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$
Fourth derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$
nth derivative:	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}[f(x)]$

Example

1) $f(x)=x^3$, find $f'''(x)$

$f'(x)=3x^2$

$f''(x)=6x$

$f'''(x)=6$

2) $\frac{dy}{dx}=5x^4-3x$, find $\frac{d^{(4)}y}{dx^{(4)}}$

$\frac{d^2y}{dx^2}=20x^3-3$

$\frac{d^3y}{dx^3}=60x^2$

$\frac{d^4y}{dx^4}=120x$

3) $y=\sin x$, find $y^{(4)}$

$y'=\cos x$

$y''=-\sin x$

$y'''=-\cos x$

$y^{(4)}=\sin x$

Acceleration

Acceleration is the derivative of velocity and the 2nd derivative of position.

Position	$x(t)$	$f(x)$
velocity	$v(t)$	$f'(x)$
Acceleration	$a(t)$	$f''(x)$

Example

The position of a particle is given by the equation $x(t) = 4t^3 - 3t^2 + 5t - 1$. Find the acceleration of the particle when $t = 3$.

$v(t)=12t^2-6t+5$

$a(t)=24t-6$

$a(3)=24(3)-6$

$a(3)=66$

Example

The velocity of a particle is given in the table below. Determine the acceleration of the particle when t = 5.

Time (sec)	0	3	4	6	9
Velocity (m/s)	4	7	10	16	17

$\frac{v(6)-v(4)}{6-4} = \frac{16-10}{6-4} = \frac{6}{2} = 3\ m/s^2$

Acceleration is the slope of the velocity function.

Product Rule

The product of two differentiable functions is differentiable itself. If *f* and *g* are differentiable, then their product *fg* is also differentiable. To find the derivative of a product:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

The derivative of a product is: "The derivative of the first function times the second plus the first times the derivative of the second function."

Examples)

1) $f(x) = x^2(2x-2)$

$f'(x) = x^2(2) + (2x-2)(2x)$

$f'(x) = 2x^2 + 4x^2 - 4x$

$f'(x) = 6x^2 - 4x$

2) $f(x) = 3x^3 \sin x$

$f'(x) = 3x^3 \cos x + \sin x(9x^2)$

$f'(x) = 3x^3 \cos x + 9x^2 \sin x$

3) $f(x) = (x^3-4)(2-3x)$

$f'(x) = (x^3-4)(-3) + (2-3x)(3x^2)$

$f'(x) = -3x^3 + 12 + 6x^2 - 9x^3$

$f'(x) = -12x^3 + 6x^2 + 12$

The product rule can be generalized for any number of products. For example,

$$\frac{d}{dx}[f(x)g(x)h(x)] = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x)$$

Example

The position of a particle is given by x(t) = $(t^2 - 4t)\cos t$. Find the acceleration of the particle when t = $\frac{\pi}{2}$.

$x(t) = (t^2-4t)\cos t$

$v(t) = (t^2-4t)(-\sin t) + \cos t(2t-4)$

$a(t) = (t^2-4t)(-\cos t) + (-\sin t)(2t-4) + (\cos t)(2) + (2t-4)(-\sin t)$

$a(\frac{\pi}{2}) = 0 + (-1)(2(\frac{\pi}{2})-4) + 0 + (2(\frac{\pi}{2})-4)(-1)$

$a(\frac{\pi}{2}) = -(\pi-4) - (\pi-4) = \boxed{-2\pi+8}$