2.3 Quotient Rule and Trig Derivatives

Pg 125 # 7, 9, 11, 40, 43-46, 65, 75, 81b, 82b, 86, 95, 98, 103-106, 108

7) $f(x)=\frac{x}{x^2+1}$

$f'(x)=\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}$

$f'(x)=\frac{x^2+1-2x^2}{(x^2+1)^2}$

$\boxed{f'(x)=\frac{1-x^2}{(x^2+1)^2}}$

9) $h(x)=\frac{x^{1/2}}{x^3+1}$

$h'(x)=\frac{(x^3+1)(\frac{1}{2}x^{-1/2})-x^{1/2}(3x^2)}{(x^3+1)^2}$

$h'(x)=\frac{\frac{1}{2}x^{5/2}+\frac{1}{2}x^{-1/2}-3x^{5/2}}{(x^3+1)^2}$

$\boxed{h'(x)=\frac{-5/2\,x^{5/2}+\frac{1}{2}x^{-1/2}}{(x^3+1)^2}}$

11) $g(x)=\frac{\sin x}{x^2}$

$g'(x)=\frac{x^2\cos x-\sin x(2x)}{(x^2)^2}$

$\boxed{g'(x)=\frac{x^2\cos x-2x\sin x}{x^4}}$

40) $f(\theta)=(\theta+1)\cos\theta$

$f'(\theta)=(\theta+1)(-\sin\theta)+\cos\theta(1)$

$\boxed{f'(\theta)=-\theta\sin\theta-\sin\theta+\cos\theta}$

43) $f(x)=-x+\tan x$

$\boxed{f'(x)=-1+\sec^2 x}$

44) $y=x+\cot x$

$\boxed{y'=1+\csc^2 x}$

45) $g(t)=t^{1/4}+6\csc t$

$g'(t)=\frac{1}{4}t^{-3/4}-6\csc t\cot t$

$\boxed{g'(t)=\frac{1}{4t^{3/4}}-6\csc(t)\cot(t)}$

46) $h(x)=x^{-1}-12\sec x$

$h'(x)=-x^{-2}-12\sec x\tan x$

$\boxed{h'(x)=\frac{-1}{x^2}-12\sec x\tan x}$

65) $f(x)=\frac{x}{x+4}$

$f'(x)=\frac{(x+4)(1)-x(1)}{(x+4)^2}$

$f'(x)=\frac{x+4-x}{(x+4)^2}$

$f'(x)=\frac{4}{(x+4)^2}$

$f'(-5)=\frac{4}{(-5+4)^2}=4$

$\boxed{y-5=4(x+5)}$

75) $f(x)=\frac{x^2}{x-1}$

$f'(x)=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$

$f'(x)=\frac{2x^2-2x-x^2}{(x-1)^2}$

$f'(x)=\frac{x^2-2x}{(x-1)^2}$

$x^2-2x=0$

$x(x-2)=0$

$x=0$ and $x=2$

$\boxed{(0,0)\text{ and }(2,4)}$

81b) $q(x) = \frac{f(x)}{g(x)}$

$$q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$q'(4) = \frac{(3)(-1) - (7)(0)}{(3)^2}$$

$$\boxed{q'(4) = \frac{-3}{9} = -\frac{1}{3}}$$

82b) $q'(7) = \frac{(4)(2) - (4)(-1)}{(4)^2}$

$$q'(7) = \frac{8+4}{16}$$

$$\boxed{q'(7) = \frac{3}{4}}$$

86) $P(t) = 500 + \frac{2000t}{50+t^2}$

$$P'(t) = \frac{(50+t^2)(2000) - (2000t)(2t)}{(50+t^2)^2}$$

$$P'(2) = \frac{(54)(2000) - (4000)(4)}{(54)^2}$$

$$\boxed{P'(2) = \frac{23000}{729} \approx 31.550}$$

95) $f(x) = \frac{x}{x-1}$

$$f'(x) = \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$f'(x) = \frac{-1}{x^2-2x+1}$$

$$f''(x) = \frac{(x^2-2x+1)(0) - (-1)(2x-2)}{(x-1)^4}$$

$$\boxed{f''(x) = \frac{2x-2}{(x-1)^4} = \frac{2}{(x-1)^3}}$$

98) $f(x) = \sec x$

$$f'(x) = \sec x \tan x$$

$$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$

$$\boxed{f''(x) = \sec^3 x + \sec x \tan^2 x}$$

103) $f(x) = 2g(x) + h(x)$

$$f'(x) = 2g'(x) + h'(x)$$

$$f'(2) = 2(-2) + (4)$$

$$\boxed{f'(2) = 0}$$

104) $f(x) = 4 - h(x)$

$$f'(x) = -h'(x)$$

$$f'(2) = -h'(2)$$

$$\boxed{f'(2) = -4}$$

105) $f(x) = \frac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{h(x)^2}$$

$$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2}$$

$$f'(2) = \frac{2-12}{1}$$

$$\boxed{f'(2) = -10}$$

106) $f(x) = g(x)h(x)$

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f'(2) = (3)(4) + (-1)(-2)$$

$$f'(2) = 12 + 2$$

$$\boxed{f'(2) = 14}$$

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