Calculus Section 2.3 Quotient Rule and Trig Derivatives

-Find the derivative using the quotient rule
-Memorize and know the trig derivatives

Homework: page 125 #'s 7, 9, 11, 40, 43-46, 65, 75, 81b, 82b, 86, 95, 98, 103-106, 108.

Quotient Rule

When two differentiable functions are divided, the resulting function is also differentiable. Let *f* and *g* be differentiable functions, then $f(x)=\frac{f}{g}$ is also differentiable. Moreover, the derivative is:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2},\ g(x)\neq 0$$

This derivative is said: "Low d-high, minus High d-low, over the square of what's below." The saying rhymes to help us remember the derivative equation.

Examples)

1) $y=\frac{5x-2}{x^2+1}$

$y'=\frac{(x^2+1)(5)-(5x-2)(2x)}{(x^2+1)^2}$

$y'=\frac{5x^2+5-10x^2+4x}{(x^2+1)^2}$

$y'=\frac{-5x^2+4x+5}{(x^2+1)^2}$

2) $f(x)=\frac{3-(1/x)}{x+5}$

$f'(x)=\frac{(x+5)\left(\frac{1}{x^2}\right)-\left(3-\frac{1}{x}\right)(1)}{(x+5)^2}$

$f'(x)=\frac{\frac{1}{x}+\frac{5}{x^2}-3+\frac{1}{x}}{(x+5)^2}$

$f'(x)=\frac{\frac{5}{x^2}+\frac{2}{x}-3}{(x+5)^2}$

3) $f(x)=\tan(x)$

$f(x)=\frac{\sin x}{\cos x}$

$f'(x)=\frac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}$

$f'(x)=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$

$f'(x)=\frac{1}{\cos^2 x}=\sec^2 x$

Trigonometric Function Review

$$\frac{1}{\sin x} = \csc x$$

$$\frac{1}{\cos x} = \sec x$$

$$\frac{1}{\tan x} = \cot x$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\csc x}{\sec x}$$

$$\sin^2 x + \cos^2 x = 1$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

Examples)

1) $y = 2x - \tan x$

$y' = x - \sec^2 x$

2) $f(x) = \dfrac{1-\csc x}{\sec x}$

$f'(x) = \dfrac{(\sec x)(\csc x \cot x) - (1-\csc x)(\sec x \tan x)}{\sec^2 x}$

$f'(x) = \dfrac{\csc x \cot x - \tan x + \csc x \tan x}{\sec x}$

3) $y = \sin x \tan x$

$y' = \sin x(\sec^2 x) + \tan x(\cos x)$