Calculus Section 2.4 The Chain Rule

-Find the value of a composite function.
-Find the derivative of a composite function using the chain rule.

Homework: Page 136 #'s 7, 9, 11, 13, 19, 23, 33, 43-49, 85, 93, 127, 128.

A **composite function** is a function such that y = f(g(x)), where the function f(x) is dependent upon the value of the function g(x). (i.e. y = sin(4x), $y = (x-2)^4$, $y = e^{5x}$)

The derivative of a composite function is found using the chain rule.

The Chain Rule

If *y = f(u)* is a differentiable function of *u* and *u = g(x)* is a differentiable function of x, then *y = f(g(x))* is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Remember this saying, "The derivative of the outside times the derivative of the inside."

Examples)

1) Find $\frac{dy}{dx}$ for $y = (x^2+1)^3$

$y' = 3(x^2+1)^2(2x)$

$y' = 6x(x^2+1)^2$

2) Find $\frac{dy}{dx}$ $f'(x)$ for $f(x) = \sqrt{x^2-1}$ $\rightarrow f(x) = (x^2-1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x$

$f'(x) = \frac{x}{\sqrt{x^2-1}}$

3) Find g'(t) when $g(t) = \frac{-7}{(2t-3)^2}$

$g(t) = -7(2t-3)^{-2}$

$g'(t) = 14(2t-3)^{-3}(2)$

$g'(t) = \frac{28}{(2t-3)^3}$

4) Find f'(x) for $f(x) = \frac{1}{3x^2+4}$ $= (3x^2+4)^{-1}$

$f'(x) = -1(3x^2+4)^{-2}(6x)$

$f'(x) = \frac{-6x}{(3x^2+4)^2}$

5) Find y' for $y=(\cos x)^2$

$y' = 2(\cos x)(-\sin x)$

$y' = -2\sin x\cos x$

6) Find y' for $y=\cos(3x^2)$

$y' = -\sin(3x^2)(6x)$

$y' = -6x\sin(3x^2)$

7) Find y' for $y=5x^2\sec(3x)$

$y' = 5x^2\sec(3x)\tan(3x)\cdot(3) + \sec(3x)(10x)$

$y' = 15x^2\sec(3x)\tan(3x) + 10x\sec(3x)$

8) Find y' for $y=\csc(2x+3)^4$

$y' = -\csc(2x+3)^4\cot(2x+3)^4 \cdot (4(2x+3)^3)\cdot(2)$

$y' = -8\csc(2x+3)^4\cot(2x+3)^4\cdot(2x+3)^3$

9) Given $g(x)=\sin(x^2)$, find g''(x).

$g'(x) = \cos(x^2)\cdot 2x$

$g''(x) = \cos(x^2)(2) + 2x(-\sin(x^2)\cdot(2x))$

$g''(x) = 2\cos(x^2) - 4x^2\sin(x^2)$