Calculus Section 2.4 Chain Rule Part II, Absolute Value

-Find the derivative using the chain rule in conjunction with the product and quotient rules.
-Find the derivative using the chain rule multiple times.
-Determine the derivative of absolute value functions.

Homework: Page 136 #'s 55, 63, 103, 104, 119-122, 125, 126

Repeated Application of the Chain Rule
In order to differentiate certain functions, the chain rule may have to be applied more than one time.

Examples)

1) $y = \sin^3 4t$

$y = (\sin(4t))^3$

$y' = 3(\sin(4t))^2 \cdot \cos(4t) \cdot 4$

$\boxed{y' = 12\sin^2(4t)\cos(4t)}$

2) $y = (\sin 2x + 4x^2)^4$

$y' = 4(\sin 2x + 4x^2)^3 \cdot (\cos 2x \cdot 2 + 8x)$

$\boxed{y' = 4(\sin 2x + 4x^2)^3 \cdot (2\cos 2x + 8x)}$

The Chain Rule Combined with Other Derivative Rules

Examples)

3) $f(x) = (2x-1)^2 \csc 2x$

$f'(x) = (2x-1)^2 \cdot [(-\csc 2x \cot 2x) \cdot 2] + \csc 2x \cdot [2(2x-1) \cdot 2]$

$\boxed{f'(x) = -2(2x-1)^2 \csc 2x \cot 2x + 4\csc 2x \cdot (2x-1)}$

Absolute Value Functions

The trick for finding the derivative of an absolute value function is to realize that $|u| = \sqrt{u^2}$. Chain rule takes over after you substitute.

Ex) Determine: $\frac{d}{dx}[|x^2 - 4|]$ $\quad |u| = |x^2 - 4|$

$$\frac{d}{dx}\left[\sqrt{(x^2-4)^2}\right]$$

$$\frac{d}{dx}\left[\left((x^2-4)^2\right)^{1/2}\right]$$

$$\frac{1}{2}\left((x^2-4)^2\right)^{-1/2} \cdot 2(x^2-4) \cdot 2x$$

$$\frac{x^2-4}{\sqrt{(x^2-4)^2}} \cdot 2x$$

$$\boxed{\frac{x^2-4}{|x^2-4|} \cdot 2x} \longleftarrow \text{substitute } |x^2-4| \text{ for } \sqrt{(x^2-4)^2}$$

Derivative of Absolute Value Functions

The general rule for the derivative of an absolute value function is:

$$\frac{d}{dx}[|u|] = \frac{u}{|u|} \cdot du$$

Ex) Write the equation of the line tangent to $g(x) = |x^3 + 3x|$ when $x = 2$.

$$g'(x) = \frac{x^3+3x}{|x^3+3x|} \cdot (3x^2+3)$$

$$g(2) = |2^3 + 6|$$

$$g(2) = 14$$

$$g'(2) = \frac{14}{14} \cdot (17)$$

$$g'(2) = 17$$

$$\boxed{y - 14 = 17(x-2)}$$