2.5 – 2.6 AP Questions

Name: Answer Key

2005 Form B AP Calculus Free-Response Questions (Non-Calc)

5. Consider the curve given by $y^2 = 2 + xy$.

   (a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

   (b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

   (d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

a) $2y\frac{dy}{dx} = x\frac{dy}{dx} + y(1)$

$\frac{dy}{dx}(2y - x) = y$

$\boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$

b) $\frac{1}{2} = \frac{y}{2y-x}$

$2y - x = 2y$

$-x = 0$

$x = 0$

$y^2 = 2 + 0$

$y = \pm\sqrt{2}$

$\boxed{(0,\sqrt{2})\ (0,-\sqrt{2})}$

c) $0 = \frac{y}{2y-x}$

This could only happen at $y = 0$. But, y cannot equal 0 because $0^2 \neq 2 + x(0)$

d) $2y\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

$3^2 = 2 + 3x$

$9 = 2 + 3x$

$7 = 3x$

$x = 7/3$

$2(3)(6) = \frac{7}{3}(6) + 3\frac{dx}{dt}$

$36 = 14 + 3\frac{dx}{dt}$

$22 = 3\frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = \frac{22}{3}}$

2004 AP Calculus Free-Response Question (Non-Calc)

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b).

$2x + 8y\frac{dy}{dx} = 3x\frac{dy}{dx} + y(3)$

$\frac{dy}{dx}(8y - 3x) = 3y - 2x$

$\boxed{\frac{dy}{dx} = \frac{3y-2x}{8y-3x}}$

b) $0 = \frac{3y-2x}{8y-3x}$

$0 = 3y - 2x$

$2x = 3y$

$2(3) = 3y$

$2 = y$

$\boxed{(3,2)}$

c) $\frac{d^2y}{dx^2} = \frac{(8y-3x)(3\frac{dy}{dx} - 2) - (3y-2x)(8\frac{dy}{dx} - 3)}{(8y-3x)^2}$

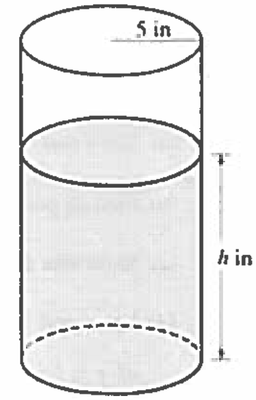
$\frac{d^2y}{dx^2} = \frac{(16-9)(0-2) - (6-6)(0-3)}{(16-9)^2}$

$\frac{d^2y}{dx^2} = \frac{7(-2)}{49}$

$\boxed{\frac{d^2y}{dx^2} = \frac{-2}{7}}$

2003 AP Calculus Free-Response Question (Non-Calc)

5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

$\frac{dr}{dt} = 0$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h\left(2\pi r \frac{dr}{dt}\right)$

$-5\pi\sqrt{h} = \pi(25)\frac{dh}{dt}$

$$\boxed{\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = \frac{-\sqrt{h}}{5}}$$

2016 AP Calculus Free-Response Questions (Non-Calc)

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$\frac{d}{dx}\left[\frac{dy}{dx} = x^2 - \frac{1}{2}y\right]$

$\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right)$

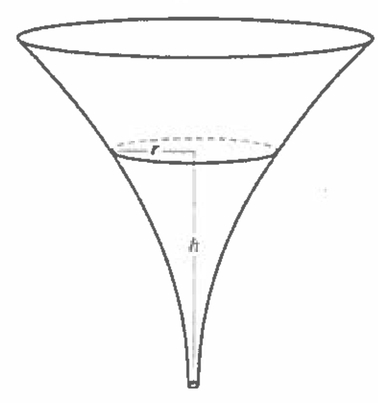
$$\boxed{\frac{dy}{dx} = 2x - \frac{1}{2}x^2 - \frac{1}{4}y}$$

2016 AP Calculus Free-Response Question (Non-calc).

The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Question 5



$r = \frac{1}{20}(3+h^2)$

$\frac{d}{dt}\left[r = \frac{3}{20} + \frac{1}{20}h^2\right]$

$\frac{dr}{dt} = \frac{1}{10}h\frac{dh}{dt}$

$-\frac{1}{5} = \frac{1}{10}(3)\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\left(\frac{10}{3}\right)$

$$\boxed{\frac{dh}{dt} = -\frac{2}{3} \text{ in/s}}$$

When the height of a cylinder is 12 cm and the radius is 4 cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min, and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing?

A. $\frac{\pi}{4}\text{cm}^3/\text{min}$

B. $4\pi\ \text{cm}^3/\text{min}$

C. $12\pi\ \text{cm}^3/\text{min}$

D. $20\pi\ \text{cm}^3/\text{min}$

E. $80\pi\ \text{cm}^3/\text{min}$

$\frac{d}{dt}[C = 2\pi r]$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$\frac{\pi}{4} = 2\pi\frac{dr}{dt}$

$\frac{1}{8} = \frac{dr}{dt}$

$\frac{dh}{dt} = 4\frac{dr}{dt}$

$\frac{dh}{dt} = 4\left(\frac{1}{8}\right)$

$\frac{dh}{dt} = \frac{1}{2}$

$\frac{d}{dt}[V = \pi r^2 h]$

$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + h\left(2\pi r\frac{dr}{dt}\right)$

$\frac{dV}{dt} = \pi(4)^2\left(\frac{1}{2}\right) + (12)\left(2\pi(4)\left(\frac{1}{8}\right)\right)$

$\frac{dV}{dt} = 8\pi + 12\pi$

$\frac{dV}{dt} = 20\pi$

Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

A) $\frac{4-3y}{2y-3x}$ B) $\frac{3x-4}{2x}$ C) $\frac{3y-x}{2}$ D) $\frac{3y-4}{2y-3x}$ E) $\frac{4+3y}{2y+3x}$

$3x\frac{dy}{dx} + y(3) = 4 + 2y\frac{dy}{dx}$

$\frac{dy}{dx}(3x - 2y) = 4 - 3y$

$\frac{dy}{dx} = \frac{4-3y}{3x-2y} = \frac{-1(3y-4)}{-1(2y-3x)} = \frac{3y-4}{2y-3x}$

The radius of a circle is decreasing at a constant rate of 0.1 centimeters per second. In terms of the circumference, C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)\pi C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2\pi C$

$\frac{d}{dt}[A = \pi r^2]$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r(-.1)$

$\frac{dA}{dt} = C(-.1)$