2.5 Implicit Differentiation

Page 145 #'s 1-7 odd, 13, 15, 29, 31, 45, 46

1) $x^2 + y^2 = 9$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

3) $x^{1/2} + y^{1/2} = 16$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$

$\frac{1}{2}y^{-1/2}\frac{dy}{dx} = -\frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = \frac{-1/2\,x^{-1/2}}{1/2\,y^{-1/2}}$

$$\boxed{\frac{dy}{dx} = -\frac{y^{1/2}}{x^{1/2}}}$$

5) $x^3 - xy + y^2 = 7$

$3x^2 - \left(x\frac{dy}{dx} + y(1)\right) + 2y\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -3x^2 + y$

$\frac{dy}{dx}(-x + 2y) = -3x^2 + y$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 + y}{-x + 2y}}$$

7) $x^3y^3 - y = x$

$x^3 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 3x^2 - \frac{dy}{dx} = 1$

$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}}$$

13) $\sin x = x(1 + \tan y)$

$\cos x = x\left(\sec^2 y\frac{dy}{dx}\right) + (1 + \tan y)(1)$

$\cos x - 1 - \tan y = x\sec^2 y\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{\cos x - 1 - \tan y}{x\sec^2 y}}$$

15) $y = \sin(xy)$

$\frac{dy}{dx} = \cos(xy) \cdot \left[x\frac{dy}{dx} + y(1)\right]$

$\frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} + y\cos(xy)$

$\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$

$$\boxed{\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}}$$

29) $(x^2+4)y = 8$

$(x^2+4)\frac{dy}{dx} + y(2x) = 0$

$\frac{dy}{dx}(x^2+4) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$

$\frac{dy}{dx} = \frac{-2(2)(1)}{(2)^2+4}$

$\frac{dy}{dx} = \frac{-4}{8}$

$\boxed{\frac{dy}{dx} = -1/2}$

31) $(x^2+y^2)^2 = 4x^2y$

$2(x^2+y^2)\cdot\left[2x + 2y\frac{dy}{dx}\right] = 4x^2\frac{dy}{dx} + y(8x)$

$(2x^2+2y^2)\left(2x+2y\frac{dy}{dx}\right) = 4x^2\frac{dy}{dx} + 8xy$

$4x^3 + 4x^2y\frac{dy}{dx} + 4xy^2 + 4y^2\frac{dy}{dx} = 4x^2\frac{dy}{dx} + 8xy$

$\frac{dy}{dx}\left(4x^2y + 4y^2 - 4x^2\right) = 8xy - 4x^3 - 4xy^2$

$\frac{dy}{dx} = \frac{8xy - 4x^3 - 4xy^2}{4x^2y + 4y^2 - 4x^2}$

$\frac{dy}{dx} = \frac{8(1)(1) - 4(1)^3 - 4(1)(1)^2}{4(1)^2(1) + 4(1)^2 - 4(1)^2}$

$\frac{dy}{dx} = \frac{8-4-4}{4+4-4}$

$\boxed{\frac{dy}{dx} = 0}$

45) $x^2 + y^2 = 4$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y}$

$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y + x\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$

$\boxed{\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}} = \frac{-y^2 - x^2}{y^3} = \frac{-(x^2+y^2)}{y^3} = \frac{-4}{y^3}$

46) $x^2y - 4x = 5$

$x^2\frac{dy}{dx} + y(2x) - 4 = 0$

$x^2\frac{dy}{dx} = 4 - 2xy$

$\frac{dy}{dx} = \frac{4-2xy}{x^2}$

$\frac{d^2y}{dx^2} = \frac{x^2\left(-2x\frac{dy}{dx} + y(-2)\right) - (4-2xy)(2x}{x^4}$

$\boxed{\frac{d^2y}{dx^2} = \frac{-2x^3\left(\frac{4-2xy}{x^2}\right) - 2x^2y - 8x + 4x^2y}{x^4}}$