Calculus Section 2.5 Implicit Differentiation

Homework: page 145 #'s 1-7 odd, 13, 15, 29, 31, 45, 46

-Distinguish between functions written in implicit form and explicit form.
-Use implicit differentiation to find the derivative of a function.

An **implicit** function is a function written where y is not isolated. Some implicit functions can be re-written in explicit form ($xy = 1$ becomes $y = \frac{1}{x}$) while others cannot ($2y^2 + 3xy = 3$).

Functions that cannot be written explicitly, must apply **implicit differentiation** to find the derivative.

When differentiating with respect to x, any term with only x variables will be differentiated normally. Any term with a y (or another variable) must be differentiated using the Chain rule because it is assumed that y is defined implicitly as a differentiable function of x.

1) $\frac{d}{dx}[x^3] =$

$3x^2$

2) $\frac{d}{dx}[y^3] =$

$3y^2 \frac{dy}{dx}$

3) $\frac{d}{dt}[V] =$

$(1)\frac{dV}{dt}$

$\frac{dV}{dt}$

3) $\frac{d}{dx}[x + 3y] =$

$1 + 3\frac{dy}{dx}$

4) $\frac{d}{dx}[xy^2] =$

$x(2y\frac{dy}{dx}) + y^2(1)$

$2xy\frac{dy}{dx} + y^2$

Steps to Implicit Differentiation

1. Differentiate both sides of the equation with respect to x.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

Find the equation for the slope of each line.

1) $y^3 + y^2 - 5y - x^2 = -4$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$$
$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$$
$$\frac{dy}{dx}(3y^2 + 2y - 5) = 2x$$
$$\boxed{\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}}$$

2) $x^3y^3 - y = x$

$$x^3\left(3y^2\frac{dy}{dx}\right) + y^3(3x^2) - \frac{dy}{dx} = 1$$
$$3x^3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$
$$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$$
$$\boxed{\frac{dy}{dx} = \frac{1-3x^2y^3}{3x^3y^2 - 1}}$$

Determine the slope of the graph: $3(x^2 + y^2)^2 = 100xy$ at the point (3,1).

$$6(x^2+y^2)\cdot\left[2x + 2y\frac{dy}{dx}\right] = 100x\frac{dy}{dx} + y(100)$$
$$12x(x^2+y^2) + 12y(x^2+y^2)\frac{dy}{dx} = 100x\frac{dy}{dx} + 100y$$
$$12y(x^2+y^2)\frac{dy}{dx} - 100x\frac{dy}{dx} = 100y - 12x(x^2+y^2)$$
$$\frac{dy}{dx}\left(12y(x^2+y^2) - 100x\right) = 100y - 12x(x^2+y^2)$$
$$\frac{dy}{dx} = \frac{100y - 12x(x^2+y^2)}{12y(x^2+y^2) - 100x}$$

$$\frac{dy}{dx} = \frac{100(1) - 12(3)(3^2+1^2)}{12(1)(3^2+1^2) - 100(3)}$$
$$\frac{dy}{dx} = \frac{100 - 36(10)}{12(10) - 300}$$
$$\frac{dy}{dx} = \frac{100-360}{120-300} = \frac{-260}{-180}$$
$$\boxed{\frac{dy}{dx} = \frac{13}{9}}$$

Find the 2nd derivative of the function 4xy = 10.

$$4x\left(\frac{dy}{dx}\right) + y(4) = 0$$
$$4x\frac{dy}{dx} = -4y$$
$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$
$$\frac{d^2y}{dx^2} = \frac{x\left(-\frac{dy}{dx}\right) - (-y)(1)}{x^2}$$
$$\frac{d^2y}{dx^2} = \frac{-x\frac{dy}{dx} + y}{x^2}$$
$$\frac{d^2y}{dx^2} = \frac{-x\left(\frac{-y}{x}\right) + y}{x^2}$$
$$\frac{d^2y}{dx^2} = \frac{y+y}{x^2}$$
$$\boxed{\frac{d^2y}{dx^2} = \frac{2y}{x^2}}$$