2.6 Related Rates Day 1

Pg. 153 #'s 11a, 13, 15a, 16a, 22, 24, 47

11a) $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(8)(4)$

$\boxed{\frac{dA}{dt} = 64\pi \text{ cm}^2/\text{min}}$

13) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(9)^2(3)$

$\frac{dV}{dt} = 972\pi \text{ in}^3/\text{min}$

b) According to the equation $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, the vol. is changing by a factor of r^2

$\frac{dV}{dt} = 4\pi(36)^2(3)$

$\frac{dV}{dt} = 15552\pi \text{ in}^3/\text{min}$

15a) $V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3(2)^2(6)$

$\boxed{\frac{dV}{dt} = 72 \text{ cm}^3/\text{s}}$

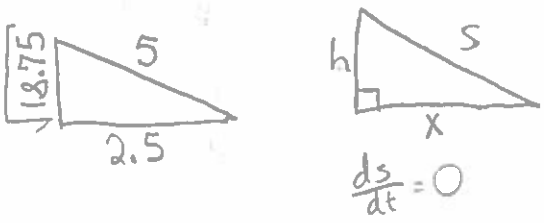
16a) $A = 6s^2$

$\frac{dA}{dt} = 12s \frac{ds}{dt}$

$\frac{dA}{dt} = 12(2)(6)$

$\boxed{\frac{dA}{dt} = 144 \text{ cm}^2/\text{s}}$

22)



$\frac{ds}{dt} = 0$

$x^2 + h^2 = s^2$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$

$2(2.5)\frac{dx}{dt} + 2\sqrt{18.75}(.15) = 2(5)(0)$

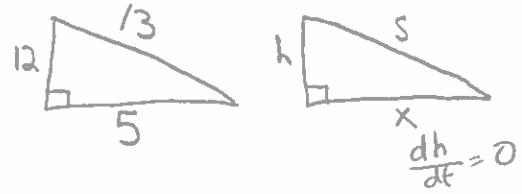
$5\frac{dx}{dt} + .3\sqrt{18.75} = 0$

$5\frac{dx}{dt} = -.3\sqrt{18.75}$

neg means moving left

$\frac{dx}{dt} = \frac{-.3\sqrt{18.75}}{5} \approx \boxed{-.260 \text{ m/s}}$

24) a)



$\frac{dh}{dt} = 0$

$x^2 + h^2 = s^2$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$

$2(5)\frac{dx}{dt} + 2(12)(0) = 2(13)(-4)$

$10\frac{dx}{dt} = -104$

$\boxed{\frac{dx}{dt} = -10.4 \text{ ft/s}}$

The speed of the boat is increasing as it gets closer to the dock.

24b)

$$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$$

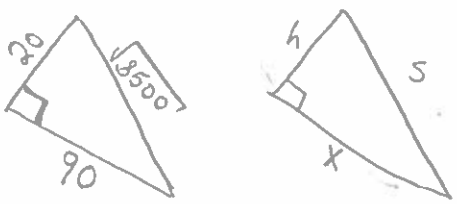
$$2(5)(-4) + 2(12)(0) = 2(13)\frac{ds}{dt}$$

$$-40 = 26\frac{ds}{dt}$$

$$\frac{ds}{dt} = -1.538 \text{ ft/s}$$

The speed of the winch decreases as the boat approaches the dock.

27)



$$x^2 + h^2 = s^2$$

$$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$$

$$2(90)(0) + 2(20)(-25) = 2\sqrt{8500}\frac{ds}{dt}$$

$$-1000 = 2\sqrt{8500}\frac{ds}{dt}$$

$$\frac{ds}{dt} = -5.423 \text{ ft/s}$$

47) $2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$ ($\frac{dh}{dt} = 0$)

$$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt}$$

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} = s\frac{ds}{dt}$$

$$x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right) = s\frac{d^2s}{dt^2} + \left(\frac{ds}{dt}\right)\left(\frac{ds}{dt}\right)$$

$$(5)\frac{d^2x}{dt^2} + (-10.4)(-10.4) = 13(0) + (-4)(-4)$$

$$(5)\frac{d^2x}{dt^2} + 108.16 = 16$$

$$\frac{d^2x}{dt^2} = \frac{16 - 108.16}{5}$$

$$\frac{d^2x}{dt^2} = -18.432 \text{ ft/s}^2$$