Calculus Section 2.6 Related Rates Part I

-Find a related rate.
-Write related rates to model real-life problems.

Homework: Page 153 #'s 11a, 13, 15a, 16a, 22, 24, 47

Real-life problems rarely involve just a single variable. Most are written in terms of multiple variables. **Related rate** problems are real-life situations based on problems that change with respect to time. We can differentiate these problems implicitly.

Volume of a Cone

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \left(\frac{1}{3}\pi r^2\right)\frac{dh}{dt} + h\left(\frac{2}{3}\pi r\frac{dr}{dt}\right)$$

$$\boxed{\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + \frac{2}{3}\pi r h\frac{dr}{dt}}$$

In General

Given the function: $y = x^2 + 3$ and $\frac{dx}{dt} = 2$ when x = 1. Find $\frac{dy}{dt}$ when x=1.

$$y = x^2 + 3$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = 2(1)(2)$$

$$\boxed{\frac{dy}{dt} = 4}$$

An Inflating Balloon

Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$4.5 = 4\pi(2)^2\frac{dr}{dt}$$

$$4.5 = 16\pi\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{4.5}{16\pi} \approx .090 \text{ ft/min}}$$

Ripples in a Pond

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

$$A = \pi r^2$$

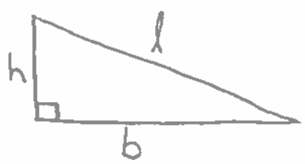
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

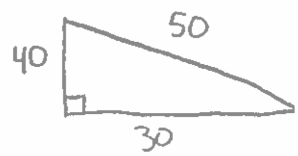
$$\frac{dA}{dt} = 2\pi(4)(1)$$

$$\boxed{\frac{dA}{dt} = 8\pi \ ft^2/s}$$

Falling Ladder

A 50 foot ladder is leaning up against the side of a building. The base of the ladder is pulled away from the wall of the building at a rate of 3 feet per second. How fast is the top of the ladder falling when the base is 30 ft. from the wall? What is the acceleration of the top of the ladder at the same point?





$$b^2 + h^2 = l^2$$

$$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 2l\frac{dl}{dt}$$

$$2(30)(3) + 2(40)\frac{dh}{dt} = 2(50)(0)$$

$$180 + 80\frac{dh}{dt} = 0$$

$$80\frac{dh}{dt} = -180$$

$$\frac{dh}{dt} = \frac{-180}{80}$$

$$\boxed{\frac{dh}{dt} = -\frac{9}{4} = -2.25 \ ft/s}$$

$$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 2l\frac{dl}{dt}$$

$$b\frac{db}{dt} + h\frac{dh}{dt} = l\frac{dl}{dt}$$

$$b\frac{d^2b}{dt^2} + \left(\frac{db}{dt}\right)\left(\frac{db}{dt}\right) + h\frac{d^2h}{dt^2} + \left(\frac{dh}{dt}\right)\left(\frac{dh}{dt}\right) = l\frac{d^2l}{dt^2} + \left(\frac{dl}{dt}\right)^2$$

$$30\frac{d^2b}{dt^2} + (3)(3) + 40\frac{d^2h}{dt^2} + \left(\frac{-9}{4}\right)\left(\frac{-9}{4}\right) = 50\frac{d^2l}{dt^2} + (0)^2$$

$$\frac{d^2b}{dt^2} = 0 \text{ and } \frac{d^2l}{dt^2} = 0$$

$$9 + 40\frac{d^2h}{dt^2} + \frac{81}{16} = 0$$

$$\boxed{\frac{d^2h}{dt^2} = \frac{-81/16 - 9}{40} \approx -.352 \ ft/s^2}$$