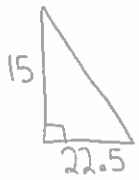
2.6 Related Rates Day 2

Pg. 154 #'s 17, 18, 20, 21b, 25, 29, 39

17)

$d = 3h$

$r = 1.5h$



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (1.5h)^2 h$

$V = \frac{2.25\pi}{3} h^3$

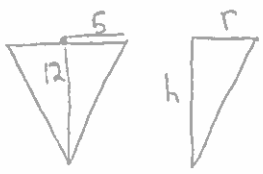
$\frac{dV}{dt} = 2.25\pi h^2 \frac{dh}{dt}$

$10 = 2.25\pi (15)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{2.25\pi (15)^2}$

$\frac{dh}{dt} \approx 0.00629$ ft/min

18)



$\frac{5}{12} = \frac{r}{h}$

$\frac{5}{12} h = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{5}{12}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{25}{144} h^3$

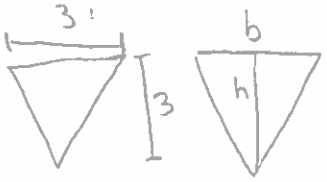
$\frac{dV}{dt} = \frac{25\pi}{144} h^2 \frac{dh}{dt}$

$10 = \frac{25\pi}{144}(8)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{\frac{25\pi}{144} 8^2}$

$\frac{dh}{dt} = 0.286$ ft/min

20)



$b = h$

a)

$V = \frac{1}{2} bh(12)$

$V = 6bh$

$V = 6h^2$

$\frac{dV}{dt} = 12h \frac{dh}{dt}$

$2 = 12(1) \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{6}$ ft/min

b) $\frac{dV}{dt} = 12h \frac{dh}{dt}$

$\frac{dV}{dt} = 12(2)\left(\frac{1}{32}\right)$

$\frac{dV}{dt} = \frac{3}{4}$ ft^3/min

$\frac{3}{8}$ in/min $= \frac{1}{32}$ ft/min

21)

b) $A = \frac{1}{2} bh$

$\frac{dA}{dt} = \frac{1}{2} b \frac{dh}{dt} + h\left(\frac{1}{2}\frac{db}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}(7)\left(\frac{-7}{12}\right) + 24\left(\frac{1}{2}(2)\right)$

$\frac{dA}{dt} = 21.958$ ft^2/s

c) $\sin\theta = \frac{b}{l}$

$\cos\theta = \frac{h}{l}$

$\cos\theta \frac{d\theta}{dt} = \frac{l\frac{db}{dt} - b\frac{dl}{dt}}{l^2}$

$\frac{24}{50} \frac{d\theta}{dt} = \frac{50(2) - 7(0)}{50^2}$

$b^2 + h^2 = l^2$

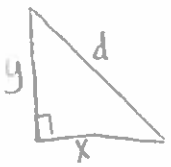
$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 2l\frac{dl}{dt}$

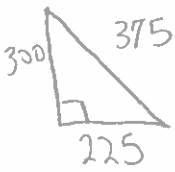
$2(7)(2) + 2(24)\frac{dh}{dt} = 0$

$\frac{dh}{dt} = \frac{-28}{48} = \frac{-7}{12}$ ft/s

$\frac{d\theta}{dt} = \frac{100}{50^2} \cdot \frac{50}{24}$

$\frac{d\theta}{dt} = \frac{1}{12}$ rad/s

25) 



a) $d^2 = x^2 + y^2$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$2(375)\frac{dd}{dt} = 2(225)(-450) + 2(300)(-600)$

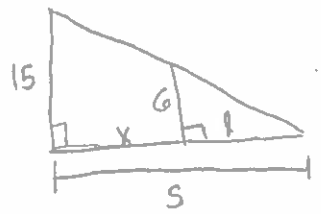
$750\frac{dd}{dt} = -202500 - 360000$

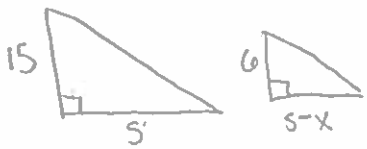
$750\frac{dd}{dt} = -562500$

$\boxed{\frac{dd}{dt} = -750 \text{ mi/hr}}$

b) $\frac{225}{450} = 1/2 \qquad \frac{300}{600} = 1/2$

The air controller has less than 30 min to correct their paths.

29) 



a) $\frac{15}{6} = \frac{s}{s-x}$

$15s - 15x = 6s$

$9s = 15x$

$s = \frac{5}{3}x$

$\frac{ds}{dt} = \frac{5}{3}\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{5}{3}(5)$

$\boxed{\frac{ds}{dt} = \frac{25}{3} \text{ ft/s}}$

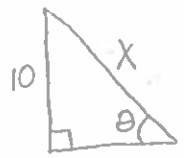
b) $l = s - x$

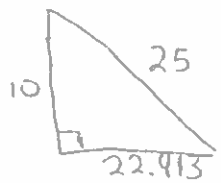
$\frac{dl}{dt} = \frac{ds}{dt} - \frac{dx}{dt}$

$\frac{dl}{dt} = \frac{25}{3} - 5$

$\frac{dl}{dt} = \frac{25}{3} - \frac{15}{3}$

$\boxed{\frac{dl}{dt} = \frac{10}{3} \text{ ft/s}}$

39) 



$\sin\theta = \frac{10}{x}$

$\sin\theta = 10x^{-1}$

$\cos\theta\frac{d\theta}{dt} = -10x^{-2}\frac{dx}{dt}$

$\cos\theta\frac{d\theta}{dt} = \frac{-10}{x^2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-10}{x^2\cos\theta}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-10}{(25)^2\left(\frac{22.913}{25}\right)}(-1)$

$\frac{d\theta}{dt} = 0.0175 \text{ rad/s}$