Answer Key

Calculus Section 2.6 Related Rates Part II

-Find a related rate.
-Use related rates to solve real-life problems.

> Homework: Page 154 #'s 17, 18, 20, 21bc, 25, 29, 39

Related Rates with Ratios

A conical paper cup 3 in. across the top and 4 in. deep is full of water. The cup springs a leak at the bottom and loses water at a constant rate of 2 cubic inches per minute. How fast is the water level dropping at the instant the water is 3 inches deep? How fast is the radius of the cup changing when the water level is 2 inches?

$\frac{r}{h} = \frac{1.5}{4}$

$r = \frac{1.5h}{4}$

$r = \frac{3}{8}h$

$h = \frac{8}{3}r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{3}{8}h\right)^2 h$

$V = \frac{3\pi}{64} h^3$

$\frac{dV}{dt} = \frac{9\pi}{64} h^2 \frac{dh}{dt}$

$-2 = \frac{9\pi}{64}(3)^2 \frac{dh}{dt}$

$-2 = \frac{81\pi}{64} \frac{dh}{dt}$

$\frac{-2(64)}{81\pi} = \boxed{\frac{dh}{dt} \approx -0.503 \text{ in/min}}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{8}{3}r\right)$

$V = \frac{8\pi}{9} r^3$

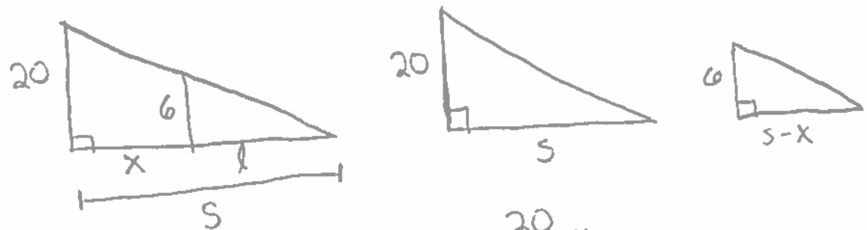
$\frac{dV}{dt} = \frac{24}{9}\pi r^2 \frac{dr}{dt}$

$-2 = \frac{24\pi}{9}(2)^2 \frac{dr}{dt}$

$-2 = \frac{96\pi}{9} \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{-2(9)}{96\pi} \approx -0.060 \text{ in/min}}$

A six ft. tall man walks at a rate of 3 feet per second toward a light stand that is 20 feet above the ground. When he is 8 feet from the base of the light, at what rate is the tip of his shadow moving? At the same point, what rate is the length of his shadow changing?



$\frac{20}{6} = \frac{s}{s-x}$

$20s - 20x = 6s$

$14s = 20x$

$s = \frac{20}{14}x$

$\frac{ds}{dt} = \frac{20}{14}\frac{dx}{dt}$

$\boxed{\frac{ds}{dt} = \frac{20}{14}(-3) = \frac{-60}{14} = \frac{-30}{7} \text{ ft/s}}$

$l = s - x$

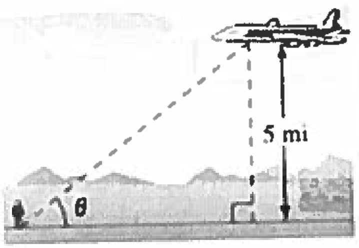
$\frac{dl}{dt} = \frac{ds}{dt} - \frac{dx}{dt}$

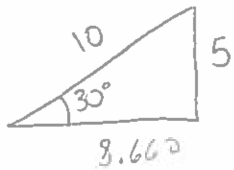
$\frac{dl}{dt} = \frac{-30}{7} - 3$

$\boxed{\frac{dl}{dt} = \frac{-51}{7} \text{ ft/s}}$

A Changing Angle

An airplane files at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rates at which the angle of elevation θ is changing when the angle is θ = 30°?





$\tan 30 = \frac{5}{x}$

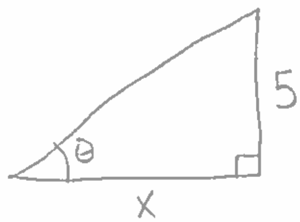
$x = \frac{5}{\tan 30}$

$x = 8.660$

$\sin 30 = \frac{5}{x}$

$x = \frac{5}{\sin 30}$

$x = 10$



$\tan\theta = \frac{5}{X}$

$\tan\theta = 5x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -5x^{-2}\frac{dx}{dt}$

$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{-5}{x^2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{-5}{8.66^2}(\cos(30))^2(-600)$

$\boxed{\frac{d\theta}{dt} = 30 \ {}^{rad}/_{hr}}$