# Calculus Section 3.1 Extrema

-Understand the definition of extrema of a function on an interval.
-Understand the definition of relative extrema.
-Find extrema on a closed interval.

Homework: Page 167 #'s 11 – 25 odd, 53, 63 – 66

**Definition of Extrema**

Let *f* be defined on an interval I containing c.

1. f(c) is the **minimum of *f* on I (min)** if $f(c) \leq f(x)$ for all x in I.

2) f(c) is the **maximum of *f* on I (max)** if $f(c) \geq f(x)$ for all x in I.

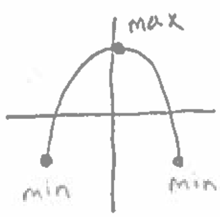
The minimum and maximum of a function on an interval are the **extreme values**, or **extrema**, of the function. The minimum and maximum of a function are also called the absolute max/min and global max/min.

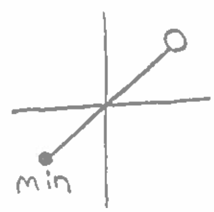
**The Extreme Value Theorem**

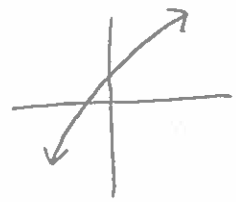
If *f* is continuous on a closed interval [a, b], then *f* has both a minimum and a maximum on the interval.

*f* has to be continuous on the interval. holes and infinity do not qualify for maximums or minimums.

**Examples)**



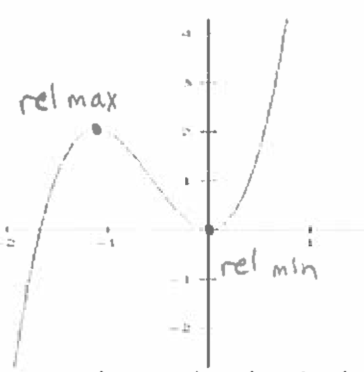



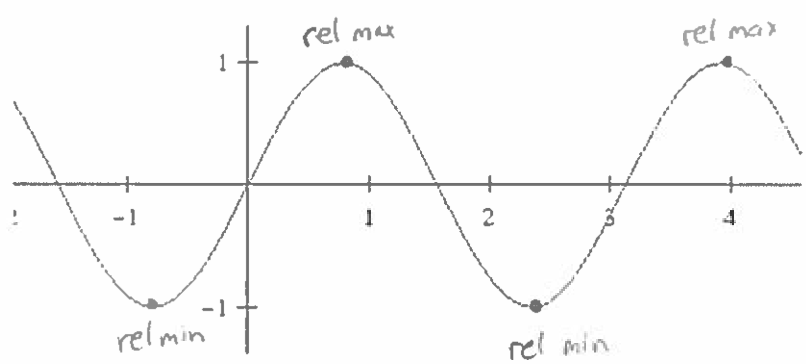



No max or min unless we restrict the domain

**Relative Extrema**

**Relative maximums** and **relative minimums** can occur inside an interval. Relative maximums are like the top of a hill and relative minimums are the bottom of a valley. There can be infinitely many relative maximums and relative minimums on an interval, but there is only one value for the absolute max and absolute min.

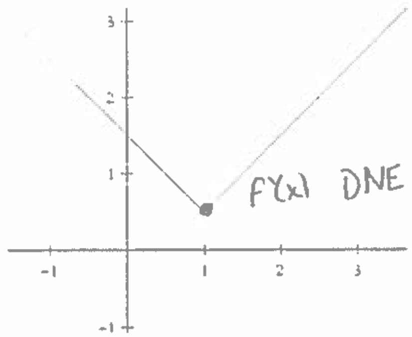


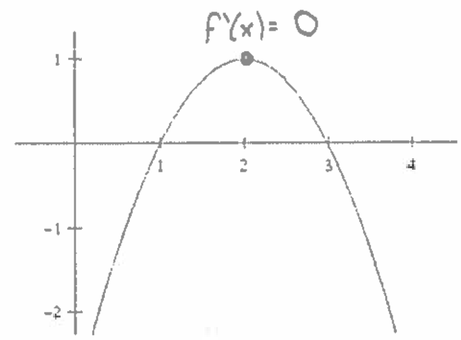




A relative extrema can be equal to the absolute extrema.

**Definition of Critical Numbers**

Let $f$ be defined at $c$. If $f'(c) = 0$ or if $f$ is not differentiable at c, then c is a **critical number** of $f$.




Relative extrema can only occur at critical numbers. Not all critical numbers will have extrema.

If $f$ has a relative minimum or maximum at x = c, then $c$ is a critical number of $f$.

**Finding Extrema on a Closed Interval**

The absolute minimum and maximum of a function will be found at either critical numbers or the endpoints of the interval. Always check all values.

**Example)** Find the absolute max/min of $f(x) = 3x^4 - 4x^3$ on the interval [-1, 2].

$f'(x) = 12x^3 - 12x^2$

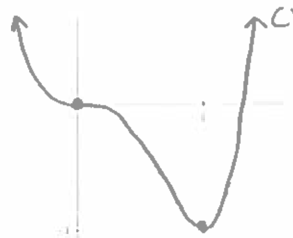
$0 = 12x^2(x-1)$

$12x^2 = 0 \qquad x-1=0$

$x=0 \qquad x=1$ critical numbers

$f(-1)=7 \quad f(0)=0 \quad f(1)=-1 \quad f(2)=16$

max $(2,16)$ min $(1,-1)$



To the left is the graph of $f(x) = 3x^4 - 4x^3$. Notice that the critical point zero does not yield a relative maximum or minimum. So, critical numbers do not have to produce extrema.

**Example**

Find the absolute max/min of $f(x) = 2x - 3x^{2/3}$ on the interval [-1, 3].

$f'(x) = 2 - 2x^{-1/3}$

$0 = 2 - \frac{2}{\sqrt[3]{x}}$ crit. point $x=0$

$\frac{2}{\sqrt[3]{x}} = 2$

$1 = \sqrt[3]{x}$

$x=1$

$f(-1)=-5 \quad f(0)=0$

$f(1)=-1 \quad f(3) = 6 - 3\sqrt[3]{9} \approx -.240$

min $(-1,-5)$ max $(0,0)$

**Example**

Find the absolute max/min of $f(x)=(\cos x)^2+\sin x$

$f(x) = \cos^2 x + \sin x$ on [0, 2π].

$f'(x) = 2\cos x(-\sin x) + \cos x$

$0 = \cos x(-2\sin x + 1)$

$\cos x = 0 \qquad -2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f(0)=1 \quad f(\pi/6)=1.25 \quad f(\frac{\pi}{2})=1$

$f(5\pi/6)=1.25 \quad f(\frac{3\pi}{2})=-1 \quad f(2\pi)=1$

max $(\pi/6, 1.25)(\frac{5\pi}{6}, 1.25)$ min $(\frac{3\pi}{2}, -1)$