Rolle's Theorem and Mean Value Theorem

Name: Answer Key

1)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

a) $h(x)$ is differentiable, and therefore continuous, on the interval. B/c $h(3) < -5 < h(1)$, the Intermediate Value Theorem guarantees $h(r) = -5$ on the interval.

a) $h(3) = f(g(3)) - 6$

$h(3) = f(4) - 6$

$h(3) = -1 - 6$

$h(3) = -7$

$h(1) = f(g(1)) - 6$

$h(1) = f(2) - 6$

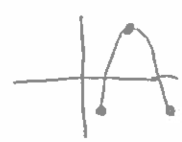
$h(1) = 9 - 6$

$h(1) = 3$

b) $\dfrac{h(3)-h(1)}{3-1} = \dfrac{-7-3}{2} = \dfrac{-10}{2} = -5$ The Mean Value Thm guarantees $h'(c) = -5$ on the interval b/c $h(c)$ is diff. and continuous.

2) **1998 #91 (AB but suitable for BC) - Calc OK:** Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

√ I. f has at least 2 zeros.
√ II. The graph of f has at least one horizontal tangent.
√ III. For some c, $2 < c < 5$, $f(c) = 3$.



a. None

b. I only

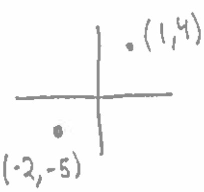
c. I and II only

d. I and III only

e. I, II and III

3) **2003 #80 (AB but suitable for BC) - Calc OK:** The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

a. There exists c, where $-2 < c < 1$, such that $f(c) = 0$. √



b. There exists c, where $-2 < c < 1$, such that $f'(c) = 0$. X

MVT

$\dfrac{4 - -5}{1 - -2} = \dfrac{9}{3} = 3$

c. There exists c, where $-2 < c < 1$, such that $f(c) = 3$. √

d. There exists c, where $-2 < c < 1$, such that $f'(c) = 3$. √

e. There exists c, where $-2 \le c \le 1$ such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$. √

Extreme Value Thm guarantees a max

4) **1998 #4 (AB but suitable for BC) - No Calc:** If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

a. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$. ✓ MVT

b. $f'(c) = 0$ for some c such that $a < c < b$. X

c. f has a minimum value on $a \le x \le b$. ✓ EVT

d. f has a maximum value on $a \le x \le b$. ✓ EVT

5) The value of c guaranteed to exist by the Mean Value Theorem for $V(x) = x^2$ in the interval [0, 3] is

x-coordinate

A) 1 B) 2 C) 3/2 D) 1/2 E) None of these

$\dfrac{V(3) - V(0)}{3 - 0} = \dfrac{9-0}{3} = 3$ $\quad V'(x) = 2x$ $\quad x = 3/2$

$3 = 2x$

6) If P(x) is continuous in [k,m] and differentiable in (k, m), then the Mean Value Theorem states that there is a point a between k and m such that

A) $\dfrac{P(k) - P(m)}{m - k} = P'(a)$

B) $P'(a)(k - m) = P(k) - P(m)$

C) $\dfrac{m - k}{P(m) - P(k)} = a$

D) $\dfrac{m - k}{P(m) - P(k)} = P'(a)$

E) None of these

$\dfrac{P(m) - P(k)}{m - k} = P'(a)$

or

$\dfrac{P(k) - P(m)}{k - m} = P'(a)$

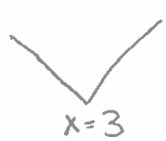
7) The Mean Value Theorem does not apply to f(x) = |x – 3| on [1,4] because

A) f(x) is not continuous on [1, 4]

B) f(x) is not differentiable on (1, 4)

C) f(1) ≠ f(4)

D) f(1) > f(4)



sharp turn → Not diff.

8) Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

$\dfrac{V(8) - V(2)}{8 - 2} = \dfrac{-120 - 100}{6} = \boxed{\dfrac{-220}{6}}$ meters/min²

(b) Do the data in the table support the conclusion that train A's velocity is −100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer. $V_A(t)$ is differentiable and continuous on the interval $5 < t < 8$. B/c $-120 \le -100 \le 40$, the Intermediate Value Thm guarantees $V_A(t) = -100$ at some point in the interval.