3.3 First Derivative Test

Page 183 #'s 9-14, 57-60, 91-96

9) $g(x) = x^2 - 2x - 8$

$g'(x) = 2x - 2$

$0 = 2x - 2$

$x = 1$

x	0	1	2
f'(x)	−	0	+

Inc: $(1, \infty)$
Dec: $(-\infty, 1)$
Rel min: $x = 1$

10) $h(x) = 12x - x^3$

$h'(x) = 12 - 3x^2$

$0 = 12 - 3x^2$

$3x^2 = 12$

$x^2 = 4$

$x = 2, -2$

x	-3	-2	0	2	3
f'(x)	−	0	+	0	−

Inc: $(-2, 2)$
Dec: $(-\infty, -2) \cup (2, \infty)$
Rel max: $x = 2$
Rel min: $x = -2$

11) $y = x(16 - x^2)^{1/2}$

$y' = x\left(\frac{1}{2}(16-x^2)^{-1/2}(-2x)\right) + (16-x^2)^{1/2}$

$y' = \frac{-x^2}{\sqrt{16-x^2}} + \sqrt{16-x^2}$

$0 = \frac{-x^2}{\sqrt{16-x^2}} + \sqrt{16-x^2}$

$-\sqrt{16-x^2} = \frac{-x^2}{\sqrt{16-x^2}}$

$-(16 - x^2) = -x^2$

$-16 + x^2 = -x^2$

$-16 = -2x^2$

$x^2 = 8$

$x = \pm 2.828$

x	-3	-2.828	0	2.828	3
f'(x)	−	0	+	0	−

Inc: $(-2.828, 2.828)$
Dec: $(-4, -2.828) \cup (2.828, 4)$
Rel max: $x = 2.828$
Rel min: $x = -2.828$

12) $y = x + 9x^{-1}$

$y' = 1 - 9x^{-2}$

$y' = 1 - \frac{9}{x^2}$

$0 = 1 - \frac{9}{x^2}$

$x = \pm 3$, DNE at $x = 0$

x	-4	-3	-1	0	1	3	4
f'(x)	+	0	−	DNE	−	0	+

Inc: $(-\infty, -3) \cup (3, \infty)$
Dec: $(-3, 0) \cup (0, 3)$
max: $x = -3$
Rel min: $x = 3$

13) $f(x) = \sin x - 1$

$f'(x) = \cos x$

$0 = \cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
f'(x)	+	0	−	0	+

Inc: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$
Dec: $(\frac{\pi}{2}, \frac{3\pi}{2})$
Rel max: $x = \frac{\pi}{2}$
Rel min: $x = \frac{3\pi}{2}$

14) $h(x) = \cos\left(\frac{x}{2}\right)$

$h'(x) = -\sin\left(\frac{x}{2}\right) \cdot \left(\frac{1}{2}\right)$

$0 = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

$0 = \sin\left(\frac{x}{2}\right)$

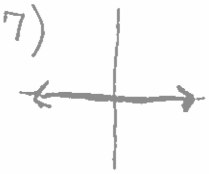
$\sin^{-1}(0) = \frac{1}{2}x$

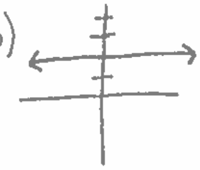
$x = 2\sin^{-1}(0)$

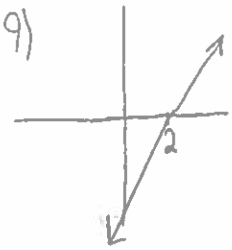
$x = 0, 2\pi$

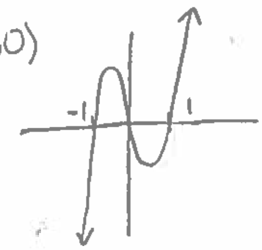
x	0	π	2π
$f(x)$	0	$-$	0

$\boxed{\text{Dec: } (0, 2\pi)}$

57) 

58) 

59) 

60) 

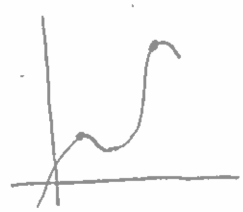
91) True

92) False, $f(x) = x$ and $g(x) = \frac{-1}{x}$,
$f(x)g(x) = -1$, $x \neq 0$

93) False, $f(x) = x^3 \rightarrow f'(x) = 3x^2$ $f'(x) = 0$ only at $x = 0$ (double root)

94) True

95) False, $f(x) = x^3$ has no rel max/min but does have a critical point at $x = 0$

96) True 

$f(x)$ must switch from decreasing after $x=1$ to increasing to get up to the point $(3,10)$