Calculus Section 3.3 The First Derivative Test

-Determine intervals on which a function is increasing or decreasing
-Apply the First Derivative Test to find relative extrema on a function

Homework: Page 186 #'s 9 – 14, 57 – 60, 91 – 96

Increasing and Decreasing Functions

Let *f* be a function that is continuous on the closed interval [a, b] and differentiable on the open interval (a, b).

1) If $f'(x) > 0$ for all x in (a, b), then *f* is **increasing** on [a, b].
2) If $f'(x) < 0$ for all x in (a, b), then *f* is **decreasing** on [a, b].
3) If $f'(x) = 0$ for all x in (a, b), then *f* is **constant** on [a, b].

Steps for Finding Increasing/Decreasing/Constant

1) Find critical numbers
2) Write intervals between those critical numbers
3) Substitute a value from each interval into $f'(x)$ to test it
4) Indicate how the function behaves from the rules above

Example)

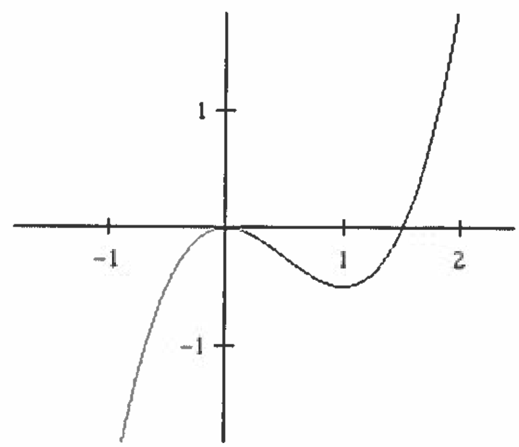
Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

$f'(x) = 3x^2 - 3x$

$0 = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0 \qquad x = 1$



x	-1	0	$1/2$	1	2
$f'(x)$	$+$	0	$-$	0	$+$

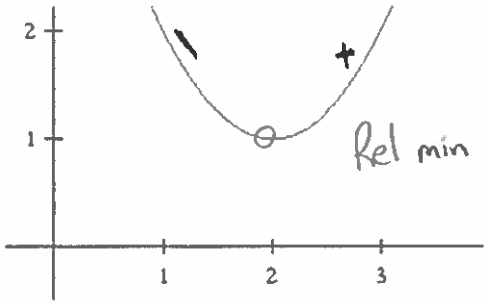
Increasing: $(-\infty, 0) \cup (1, \infty)$

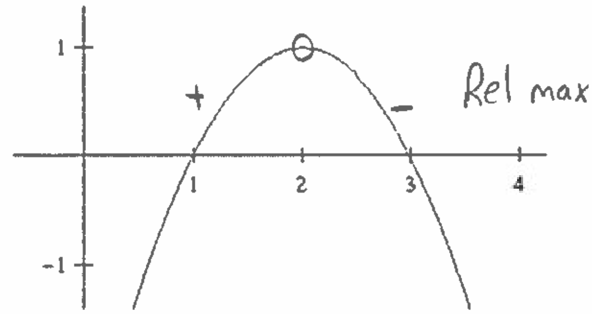
Decreasing: $(0, 1)$

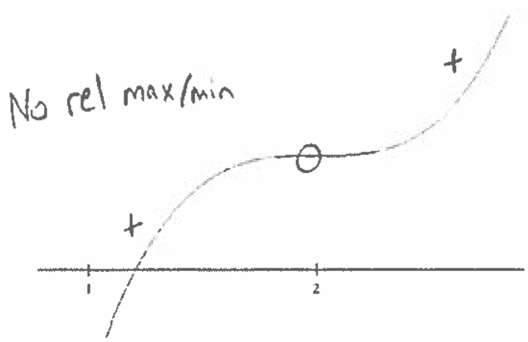
The First Derivative Test

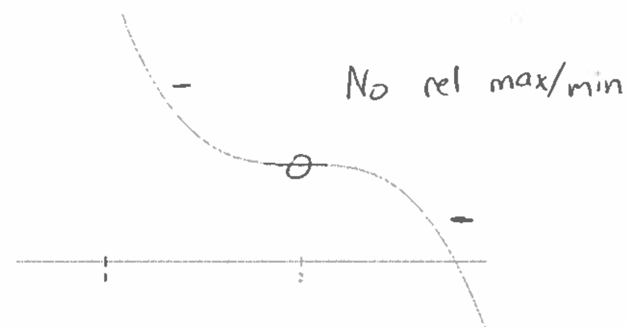
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then f(c) can be classified as follows:

1) If $f'(x)$ changes from negative to positive at c, then f has a relative minimum at (c, f(c)).
2) If $f'(x)$ changes from positive to negative at c, then f has a relative maximum at (c, f(c)).
3) If $f'(x)$ is positive on both sides of c or negative on both sides of c, then f(c) is neither a relative maximum nor a relative minimum.









Examples)

Find the relative extrema and inc./dec. intervals of

$f(x) = \frac{1}{2}x - \sin x$ on $[0, 2\pi]$.

$f'(x) = \frac{1}{2} - \cos x$

$0 = \frac{1}{2} - \cos x$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

X	$\pi/6$	$\pi/3$	π	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
$f'(x)$	−	0	+	0	−

Inc: $(\frac{\pi}{3}, \frac{5\pi}{3})$ Dec: $(0, \frac{\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi)$

Rel max: $x = \frac{5\pi}{3}$ Rel min: $x = \frac{\pi}{3}$

Find the relative extrema and inc./dec. intervals of

$f(x) = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2} = x^2 + x^{-2}$

$f'(x) = 2x - 2x^{-3}$

$f'(x) = 2x - \frac{2}{x^3}$ ← DNE at $x = 0$

$0 = 2x - \frac{2}{x^3}$

$\frac{2}{x^3} = 2x$

$2 = 2x^4$

$1 = x^4$

$x = \pm 1$

x	-2	-1	-1/2	0	1/2	1	2
$f'(x)$	−	0	+	DNE	−	0	+

Inc: $(-1, 0) \cup (1, \infty)$

Dec: $(-\infty, -1) \cup (0, 1)$

Rel max: None → function DNE at x=0

Rel min: $x = -1$ and $x = 1$