3.4 2nd Derivative Test

Pg 192 #'s 1, 2, 15, 17, 24, 33, 34, 50, 77, 78

1) $f'(x) > 0$

$f''(x) < 0$

2) $f'(x) < 0$

$f''(x) > 0$

15) $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$0 = 6x - 12$

$x = 2$

x	0	2	3
$f''(x)$	-	0	+

$\boxed{\text{concave up: } (2,\infty) \quad \text{concave down: } (-\infty, 2)}$

17) $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x$

$0 = 6x(x+2)$

$x = 0 \quad x = -2$

x	-3	-2	-1	0	1
$f''(x)$	+	0	-	0	+

$\boxed{\text{concave up: } (-\infty,-2)\cup(0,\infty) \quad \text{concave down: } (-2,0)}$

24) $f(x) = x^{1/2} + 3x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$

$f''(x) = -\frac{1}{4}x^{-3/2} + \frac{9}{4}x^{-5/2}$

$0 = -\frac{1}{4}x^{-3/2} + \frac{9}{4}x^{-5/2}$

$\frac{1}{4}x^{-3/2} = \frac{9}{4}x^{-5/2}$

$\frac{1}{\sqrt{x^3}} = \frac{9}{\sqrt{x^5}}$ ← DNE at x=0

$(\sqrt{x^5})^2 = (9\sqrt{x^3})^2$ ← Restrict to pos values of x

$x^5 = 81x^3$

$x^2 = 81$

$x = 9$

x	0	1	9	10
$f''(x)$	DNE	+	0	-

$\boxed{\text{concave up: } (0,9) \quad \text{concave down: } (9,\infty)}$

33) $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0 \quad x = 2$

$f''(x) = 6x - 6$

$f''(0) = -6 \quad f''(2) = 6$

$\boxed{\text{rel max at } x=0 \quad \text{rel min at } x=2}$

34) $f(x) = -x^3 + 7x^2 - 15x$

$f'(x) = -3x^2 + 14x - 15$

$f'(x) = -(3x^2 - 14x + 15)$

$f'(x) = -(3x-5)(x-3)$

$0 = -(3x-5)(x-3)$

$x = \frac{5}{3}$ $x = 3$

$f''(x) = -6x + 14$

$f''(\frac{5}{3}) = 4$ $f''(3) = -4$

rel min at $x = \frac{5}{3}$	rel max at $x = 3$

$\underline{45}$ $\frac{-9 \times 5}{-14}$

$(x - \frac{9}{3})(x - \frac{5}{3})$

$(x-3)(3x-5)$

50) a) $s' > 0$ $s'' > 0$

b) $s' > 0$ $s'' < 0$

c) s' ? $s'' = 0$

d) $s' = 0$ $s'' = 0$

e) $s' < 0$ $s'' > 0$

f) $s' < 0 \rightarrow s' > 0$ $s'' > 0$

77) False, an increasing function can be concave up or concave down

78) False, points of inflection only occur where the graph changes concavity