3.7 Optimization

Pg. 220 #'s 2bc, 9, 10, 19, 21, 31, 32, 37

2) $V=(24-2x)^2(x)$

$V=(576-96x+4x^2)x$

$V=4x^3-96x^2+576x$

$\frac{dV}{dx}=12x^2-192x+576$

$0=12(x^2-16x+48)$

$0=12(x-12)(x-4)$

$x=12$ $\boxed{x=4}$

$V=(24-2(4))^2(4)$

$V=(24-8)^2(4)$

$\boxed{V=1024 \text{ in}^3}$

10) $A=lw$ $P=2l+2w$

$A=(\frac{1}{2}P-w)w$ $\frac{1}{2}P=l+w$

$A=\frac{1}{2}Pw-w^2$ $l=\frac{1}{2}P-w$

$\frac{dA}{dw}=\frac{1}{2}P-2w$

$0=\frac{1}{2}P-2w$

$2w=\frac{1}{2}P$

$\boxed{w=\frac{1}{4}P,\ l=\frac{1}{4}P \quad A=\frac{1}{16}P^2}$

9) $A=lw$ $P=2l+2w$

$A=(40-w)w$ $80=2l+2w$

$A=40w-w^2$ $40=l+w$

$l=40-w$

$\frac{dA}{dw}=40-2w$

$0=40-2w$

$2w=40$

$\boxed{w=20,\ l=20 \quad A=400}$

19) $P=2y+x$ $A=xy$

$245{,}000=xy$

$x=\frac{245000}{y}$

$P=2y+\frac{245000}{y}$

$P=2y+245000y^{-1}$

$\frac{dP}{dy}=2-245000y^{-2}$

$0=2-\frac{245000}{y^2}$

$\frac{245000}{y^2}=2$

$245000=2y^2$

$122500=y^2$

$y=350$

$245000=350x$

$x=700$

$\boxed{350_m \times 700_m}$

21) $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$P = x + \frac{1}{2}\pi x + 2y$

$16 = x + \frac{1}{2}\pi x + 2y$

$y = 8 - \frac{1}{2}x - \frac{1}{4}\pi x$

$A = x\left(8 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{\pi}{8}x^2$

$A = 8x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{\pi}{8}x^2$

$\frac{dA}{dx} = 8 - x - \frac{1}{2}\pi x + \frac{\pi}{4}x$

$0 = 8 - x - \frac{1}{2}\pi x + \frac{\pi}{4}x$

$-8 = -x\left(1 + \frac{1}{2}\pi - \frac{\pi}{4}\right)$

$8 = x\left(1 + \frac{\pi}{4}\right)$

$\frac{8}{1 + \frac{\pi}{4}} = x$

$\boxed{\frac{32}{4+\pi} = x}$

$y = 8 - \frac{1}{2}\left(\frac{32}{4+\pi}\right) - \frac{\pi}{4}\left(\frac{32}{4+\pi}\right)$

$y = \frac{32}{4}\left(\frac{4+\pi}{4+\pi}\right) - \frac{64}{4}\left(\frac{1}{4+\pi}\right) - \frac{32\pi}{4}\left(\frac{1}{4+\pi}\right)$

$y = \frac{128 + 32\pi - 64 - 32\pi}{4(4+\pi)}$

$y = \frac{64}{4(4+\pi)}$

$\boxed{y = \frac{16}{4+\pi}}$

31) The volume does not remain the same. When you squeeze a bottle, the contents get pushed out because the volume inside the bottle shrinks. The shape of a squeezed-bottle no longer yields an optimized volume based on the constant surface area.

32) There is no minimum area because the area keeps decreasing for $\lim_{l \to 0} lw$ until you reach the trivial solution of the area being zero.

37) $S = kh^2w$

$20^2 = h^2 + w^2$

$h^2 = 400 - w^2$

$S = k(400 - w^2)w$

$S = k(400w - w^3)$

$S = 400kw - kw^3$

$\frac{dS}{dw} = 400k - 3kw^2$

$0 = 400k - 3kw^2$

$3kw^2 = 400k$

$w^2 = \frac{400}{3}$

$w = \frac{20}{\sqrt{3}}$

$h^2 = 400 - \left(\frac{20}{\sqrt{3}}\right)^2$

$h^2 = 400 - \frac{400}{3}$

$h^2 = \frac{800}{3}$

$h^2 = \frac{20\sqrt{2}}{\sqrt{3}}$

$\boxed{w = \frac{20\sqrt{3}}{3} \quad h = \frac{20\sqrt{6}}{3}}$