Calculus Section 3.7 Optimization

- Solve applied minimum and maximum problems

Homework: Page 220 #'s 2bc, 9, 10, 19, 21, 31, 32, 37

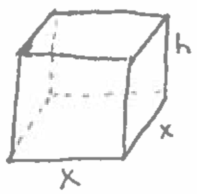
Optimization problems are an application of calculus that allows you to determine the minimum or maximum value of a problem. Some examples include: greatest profit, least time, greatest voltage, optimum size, greatest strength, and least distance.

The primary formula is the initial formula that you are trying to optimize.

A secondary equation may be needed if the primary formula cannot be written in terms of a single variable.

Example

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce the box with maximum volume?



$$V = x^2 h$$

$$V = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$V = \frac{108x^2}{4x} - \frac{x^4}{4x}$$

$$V = 27x - \frac{1}{4}x^3$$

$$\frac{dV}{dx} = 27 - \frac{3}{4}x^2$$

$$SA = x^2 + 4xh$$

$$108 = x^2 + 4xh$$

$$h = \frac{108 - x^2}{4x}$$

$$0 = 27 - \frac{3}{4}x^2$$

$$\frac{3}{4}x^2 = 27$$

$$x^2 = 36$$

$$x = 6$$

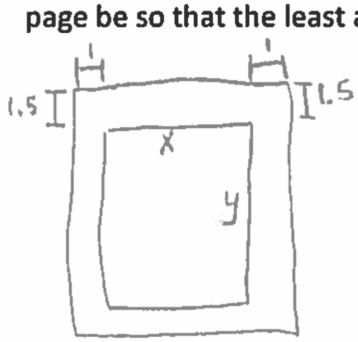
$$h = \frac{108 - (6)^2}{4(6)}$$

$$h = 3$$

$$\boxed{6_{in} \times 6_{in} \times 3_{in}}$$

Finding Minimum Area

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?



$$A = (x+2)(y+3)$$

$$A = \left(\frac{24}{y} + 2\right)(y + 3)$$

$$A = 24 + \frac{72}{y} + 2y + 6$$

$$A = 30 + 72y^{-1} + 2y$$

$$\frac{dA}{dx} = -72y^{-2} + 2$$

$$0 = \frac{-72}{y^2} + 2$$

$$A = xy$$

$$24 = xy$$

$$x = \frac{24}{y}$$

$$\frac{72}{y^2} = 2$$

$$72 = 2y^2$$

$$y^2 = 36$$

$$y = 6$$

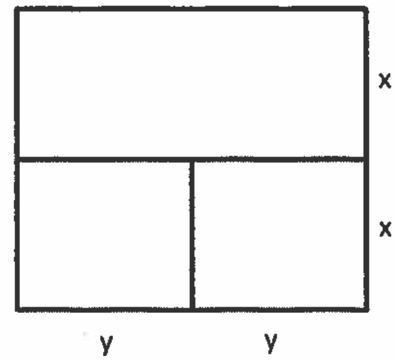
$$x = \frac{24}{6}$$

$$x = 4$$

$$\boxed{6\,in \times 9\,in}$$

Example)

A farmer has 100 yards of fencing to form two identical rectangular pens and a third pen that is twice as long as the other two pens (as shown in the diagram). All three pens have the same width, x. Which value of y produces the maximum total fenced area?



$A = (2y)(2x)$

$A = 4xy$

$A = 4(20 - \frac{6}{5}y)y$

$A = (80 - \frac{24}{5}y)y$

$A = 80y - \frac{24}{5}y^2$

$\frac{dA}{dy} = 80 - \frac{48}{5}y$

$0 = 80 - \frac{48}{5}y$

$P = 6y + 5x$

$100 = 6y + 5x$

$5x = 100 - 6y$

$x = 20 - \frac{6}{5}y$

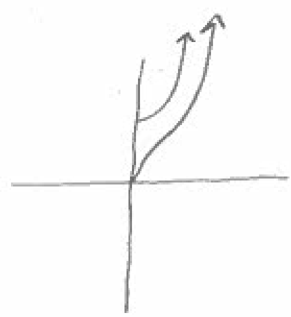
$\frac{48}{5}y = 80$

$48y = 400$

$\boxed{y = \frac{25}{3} = 8.\bar{3}}$

Example)

The position of an object moving along a straight line for t ≥ 0 is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at t = 0, at what time is the distance between the objects a minimum?



$d = s_1 - s_2$

$d = (t^3 + 2) - (t^2)$

$\frac{dd}{dt} = 3t^2 - 2t$

$0 = 3t^2 - 2t$

$0 = t(3t - 2)$

$t = 0 \quad t = 2/3$

$d(0) = 2$

$\boxed{\begin{array}{c} t = 2/3 \\ d(\frac{2}{3}) = 1.852 \end{array}}$