Calculus Section 4.1 Antiderivatives and Indefinite Integration

-Write the general solution of a differential equation.
-Use indefinite integral notation for antiderivatives.
-Use basic integration rules to find antiderivatives.

Homework: page 251 #'s 11 – 23, 69, 72, 73, 74

Antiderivatives
A function f(x) is an **antiderivative** of *g(x)* if f '(x) = g(x) for all x.

Example)
If $f(x) = 3x^2$, what is the antiderivative of f(x)?

x^3 because the derivative of x^3 is $\frac{d}{dx}[x^3] = 3x^2$

The antiderivative for the function f(x) is just <u>an</u> antiderivative not <u>the</u> antiderivative. There can be infinitely many antiderivatives for any function f(x).

Example)
What is the antiderivative of $f(x) = 4x^3$?

$x^4 + 3$ or $x^4 - 5$ or $x^4 + \pi$ or $x^4 + e$ $\boxed{x^4 + C \text{ where } C \text{ is any constant}}$

********The antiderivative <u>MUST</u> have a +C, the **constant of integration.**********

The operation of finding the antiderivative is called **antidifferentiation** or <u>indefinite integration</u>.

Integral symbol   variable of integration

$$\int \underbrace{f'(x)}_{\text{Integrand}} dx = f(x) + C$$

constant of integration

Basic Integration Rules (other examples found on p. 246 and inside the front cover of the book)

$\int 0dx =$ $0x + C = C$

$\int kdx =$ $kx + C$

$\int kf'(x)dx =$ $k\int f'(x)dx = kf(x) + C$

$\int [f'(x) + g'(x)] =$ $f(x) + g(x) + C$

$\int x^n dx =$ $\frac{x^{n+1}}{n+1}$ ← integration "power rule"

Examples)

Integrate the following functions to find the antiderivative.

1) $\int 3x\,dx$

$\boxed{\frac{3}{2}x^2 + C}$

2) $\int \frac{1}{x^2}\,dx$

$\int x^{-2}\,dx$

$\frac{x^{-1}}{-1} + C$

$\boxed{\frac{-1}{x} + C}$

3) $\int \sqrt{t}\,dt$

$\int t^{1/2}\,dt$

$\frac{t^{3/2}}{3/2} + C$

$\boxed{\frac{2}{3}t^{3/2} + C}$

4) $\int (y+2)\,dy$

$\boxed{\frac{1}{2}y^2 + 2y + C}$

5) $\int (2\theta^2 - 3\theta + 1)\,d\theta$

$\boxed{\frac{2}{3}\theta^3 - \frac{3}{2}\theta^2 + \theta + C}$

6) $\int \frac{x+1}{\sqrt{x}}\,dx$

$\int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx$

$\int \left(x^{1/2} + x^{-1/2}\right)dx$

$\boxed{\frac{2}{3}x^{3/2} + 2x^{1/2} + C}$