Calculus Sections 4.1 Particular Solutions for Integrals

-Use basic integration rules to find antiderivatives.
-Find a particular solution of an integral.

Homework: Page 251 #'s 35-42, 51, 52

The constant of integration, C, must be included any time we integrate. This is because the antiderivative is a family of solutions varying by the constant of integration. In order to find an actual value for C, you must know an **initial condition** so that your antiderivative isn't a general solution (with +C) but a **particular solution** where the constant of integration is known.

Example)

Find the particular solution that satisfies the initial condition f(1) = 0 for $f\,'(x) = \frac{1}{x^2},\ x > 0$.

$f(x) = \int \frac{1}{x^2}\,dx$

$f(x) = \int x^{-2}\,dx$

$f(x) = -1x^{-1} + C$

$f(x) = \frac{-1}{x} + C$

$0 = \frac{-1}{(1)} + C$

$0 = -1 + C$

$1 = C$

$\boxed{f(x) = \frac{-1}{x} + 1}$

Find the particular solution given the following conditions: $f\,''(x) = 2x + 3,\ f\,'(2) = 8,$ and $f(2) = 0$.

$f''(x) = 2x+3$

$f'(x) = \int (2x+3)\,dx$

$f'(x) = x^2 + 3x + C$

$8 = (2)^2 + 3(2) + C$

$-2 = C$

$f'(x) = x^2 + 3x - 2$

$f(x) = \int (x^2 + 3x - 2)\,dx$

$f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + C_2$

$0 = \frac{1}{3}(2)^3 + \frac{3}{2}(2)^2 - 2(2) + C_2$

$-\frac{14}{3} = C_2$

$\boxed{f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x - \frac{14}{3}}$