4.1 Particular Solutions for Integrals Part II

Page 252 #'s 53, 54, 67, 61 ← including speeding up/slowing down

53) $\int a(t) = \int -32$

$v(t) = -32t + C$

$60 = -32(0) + C$

$60 = C$

$\int v(t) = \int -32t + 60$

$x(t) = -16t^2 + 60t + C_2$

$6 = -16(0)^2 + 60(0) + C_2$

$6 = C_2$

$\boxed{x(t) = -16t^2 + 60t + 6}$

54) $\int a(t) = \int -32$

$v(t) = -32t + C$

$V_0 = -32(0) + C$

$V_0 = C$

$\int v(t) = \int -32t + V_0$

$x(t) = -16t^2 + V_0t + C_2$

$0 = -16(0)^2 + V_0(0) + C_2$

$0 = C_2$

$x(t) = -16t^2 + V_0t$

$550 = -16t^2 + V_0t$

$-16t^2 + V_0t - 550 = 0$

$t = \frac{-b}{2a}$ ← the vertex of the parabola

$t = \frac{-V_0}{-32} = \frac{V_0}{32}$

$-16\left(\frac{V_0}{32}\right)^2 + V_0\left(\frac{V_0}{32}\right) - 550 = 0$

$\frac{-1}{64}V_0^2 + \frac{1}{32}V_0^2 = 550$

$\frac{1}{64}V_0^2 = 550$

$V_0^2 = 35200$

$\boxed{V_0 \approx 187.617 \text{ ft/s}}$

67) $\int a(t) = \int 6$

$v(t) = 6t + C$

$0 = 6(0) + C$

$0 = C$

$\int v(t) = \int 6t$

$x(t) = 3t^2 + C_2$

$0 = 3(0)^2 + C_2$

$0 = C_2$

$x(t) = 3t^2$ } car

$\int v(t) = \int 30$

$x(t) = 30t + C_3$

$0 = 30(0) + C_3$

$0 = C_3$

$x(t) = 30t$ } truck

$3t^2 = 30t$

$3t^2 - 30t = 0$

$3t(t-10) = 0$

$t = 0 \quad t = 10$

a) car $x(10) = 3(10)^2$

$\boxed{x(10) = 300 \text{ Ft}}$

b) car $v(10) = 6(10)$

$\boxed{v(10) = 60 \text{ ft/s}}$

61) $x(t) = t^3 - 6t^2 + 9t - 2 \quad 0 \le t \le 5$

a) $v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

$v(t) = 0$

$0 = 3t^2 - 12t + 9$

$0 = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

$t = 3$ and $t = 1$

$a(t) = 0$

$6t - 12 = 0$

$6t = 12$

$t = 2$

t	$1/2$	1	2	3	4
$v(t)$	+	0	-	0	+

t	1	2	3
$a(t)$	-	0	+

moving left: $(1,3)$

b) moving right: $(0,1) \cup (3,5)$

c) $v(2) = 3(2)^2 - 12(2) + 9$

$v(2) = -3$

t	$1/2$	1	1.5	2	2.5	3	4
$v(t)$	+	0	-	-	-	0	+
$a(t)$	-	-	-	0	+	+	+

speeding up: $(1,2) \cup (3,5)$

slowing down: $(0,1) \cup (2,3)$