Calculus Section 4.3 Properties of Definite Integrals

-Evaluate a definite integral using properties of definite integrals

Page 273 #'s 13, 15, 23, 26, 39-41, 44, 47-49, 61-64

An indefinite integral is used to find the antiderivative of a function.

The definite integral is used to notate finding the area under a curve.

Definite Integral

If the function f is continuous on the closed interval [a, b], then the area of the region bounded by the graph of f and the x-axis is:

$$Area = \int_a^b f(x)dx$$

where a and b are the endpoints of the region whose area you are finding.

The value of a is always the left most (smallest value) of the interval. For instance, the area under the curve bounded by [3, 7] will have a = 3 and b = 7: $Area = \int_3^7 f(x)dx$.

The definite integral is an accumulator of area. This means that the area adds to its value as it moves from left to right.

If the value of b is smaller than the value of a, then the area is moving backward and will be negative.

$$\int_b^a f(x)dx = -\int_a^b f(x)dx \quad \text{or the put numbers to it} \quad \int_4^1 f(x)dx = -\int_1^4 f(x)dx$$

Areas above the x-axis are considered positive while areas under the x-axis are negative. This means that a negative area while moving right-to-left would be counted as positive.

Example)

$\int_0^1 f(x)dx = 2$

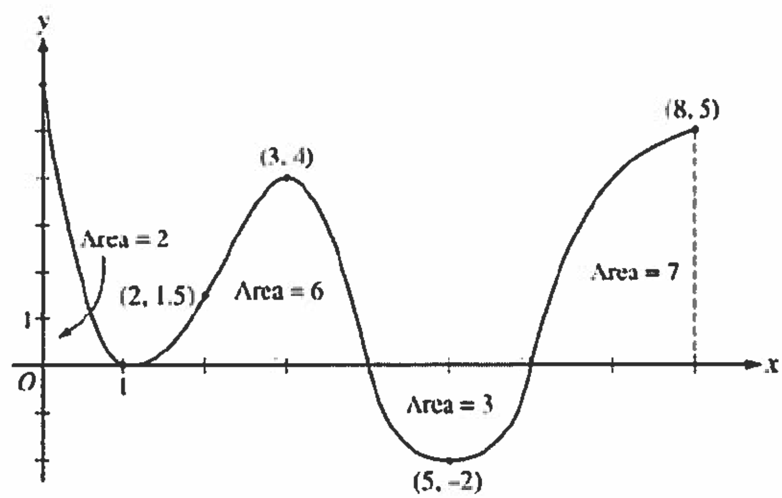
$\int_0^4 f(x)dx = 2+6=8$

$\int_1^6 f(x)dx = 6-3=3$

$\int_4^8 f(x)dx = -3+7=4$

$\int_0^8 f(x)dx = 2+6-3+7=12$

$\int_4^1 f(x)dx = -\int_1^4 f(x)dx = -6$



Graph of f'

If f(8) = 4, determine the absolute minimum value of f on the closed interval 0 ≤ x ≤ 8.

$\int_8^6 f(x)dx = -7 \qquad \int_8^4 f(x)dx = -4 \qquad \int_8^1 f(x)dx = -10 \qquad \int_8^0 f(x)dx = -12$

$4-12=-8$

$\boxed{f(0)=-8}$

Properties of Definite Integrals

1) If f is defined at x = a, then $\int_a^a f(x)dx = 0$

2) If f is integrable on the entire interval [a, b] and c is a value such that a < c < b, then
$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

3) $\int_a^b k \cdot f(x)dx = k\int_a^b f(x)dx$

4) $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

5) If f(x) is an even function, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$

6) If f(x) is an odd function, then $\int_{-a}^a f(x)dx = 0$

Example) $\int_2^4 f(x)dx = 5$ and $\int_2^6 f(x)dx = -6$. What is the value of $\int_4^6 f(x)dx$?

$$\int_2^6 f(x)dx = \int_2^4 f(x)dx + \int_4^6 f(x)dx$$

$$-6 = 5 + \int_4^6 f(x)dx$$

$$-11 = \int_4^6 f(x)dx$$

Example) $\int_1^3 x^2dx = \frac{26}{3}$, $\int_1^3 xdx = 4$, and $\int_1^3 dx = 2$. Evaluate $\int_1^3(-x^2 + 4x - 3)dx$.

$$-\int_1^3 x^2dx + 4\int_1^3 xdx - 3\int_1^3 dx$$

$$-\left(\frac{26}{3}\right) + 4(4) - 3(2)$$

$$-\frac{26}{3} + 16 - 6$$

$$\frac{4}{3}$$