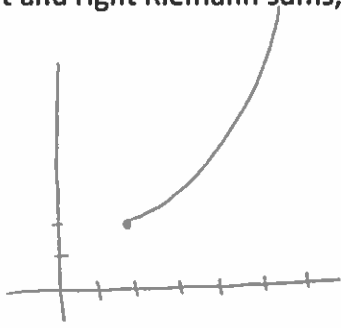


Riemann Sums Worksheet

1) Find the left and right Riemann sums, midpoint sum, and trapezoidal sum for $y = x^2 - 2$ on $[2, 6]$ with 4 even subintervals.



Left

$$A \approx f(2) \times 1 + f(3) \times 1 + f(4) \times 1 + f(5) \times 1$$

$$A \approx 2 + 7 + 14 + 23$$

$$A \approx 46$$

Right

$$A \approx f(6) \times 1 + f(5) \times 1 + f(4) \times 1 + f(3) \times 1$$

$$A \approx 34 + 23 + 14 + 7$$

$$A \approx 78$$

Midpoint

$$A \approx f(2.5) \times 1 + f(3.5) \times 1 + f(4.5) \times 1 + f(5.5) \times 1$$

$$A \approx 4.25 + 10.25 + 18.25 + 28.25$$

$$A \approx 61$$

Trapezoid

$$A \approx \frac{f(2)+f(3)}{2}(1) + \frac{f(3)+f(4)}{2}(1) + \frac{f(4)+f(5)}{2}(1) + \frac{f(5)+f(6)}{2}(1)$$

$$A \approx \frac{2+7}{2} + \frac{7+14}{2} + \frac{14+23}{2} + \frac{23+34}{2}$$

$$A \approx 62$$

2) The velocity of a particle at different times is given in the table below. Use the left and right Riemann sums, midpoint sum (2 partitions), and trapezoidal sum to approximate the position of the particle if the particle has an initial position of $x = 4$.

t	1	3	5	8	9
v(t)	0	2	8	12	22

Left

$$x(9) \approx v(1) \times 2 + v(3) \times 2 + v(5) \times 3 + v(8) \times 1 + C$$

$$x(9) \approx 0 \times 2 + 2 \times 2 + 8 \times 3 + 12 \times 1 + C$$

$$x(9) \approx 0 + 4 + 24 + 12 + 4$$

$$x(9) \approx 44$$

Right

$$x(9) \approx v(9) \times 1 + v(8) \times 3 + v(5) \times 2 + v(3) \times 2 + C$$

$$x(9) \approx 22 \times 1 + 12 \times 3 + 8 \times 2 + 2 \times 2 + C$$

$$x(9) \approx 22 + 36 + 16 + 4 + 4$$

$$x(9) \approx 82$$

Midpoint

$$x(9) \approx v(3) \times 4 + v(8) \times 4 + C$$

$$x(9) \approx 2 \times 4 + 12 \times 4 + C$$

$$x(9) \approx 8 + 48 + 4$$

$$x(9) \approx 60$$

Trapezoid

$$x(9) \approx \frac{v(1)+v(3)}{2} (2) + \frac{v(3)+v(5)}{2} (2) + \frac{v(5)+v(8)}{2} (3) + \frac{v(8)+v(9)}{2} (1) + C$$

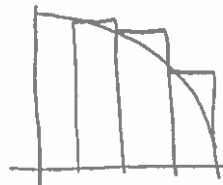
$$x(9) \approx \frac{0+2}{2} (2) + \frac{2+8}{2} (2) + \frac{8+12}{2} (3) + \frac{12+22}{2} (1) + C$$

$$x(9) \approx 2 + 10 + 30 + 17 + 4$$

$$x(9) \approx 63$$

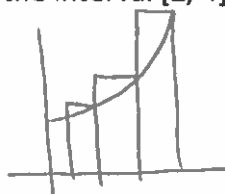
True or False

3) The function $f(x)$ is decreasing on the interval $[1, 5]$. The left Riemann sum to approximate the area under the curve is an overestimate.



True

4) The function $g(x)$ is increasing on the interval $[2, 4]$. The right Riemann sum to approximate the area under the curve is an underestimate.



False