Calculus Section 4.3 Riemann Sums

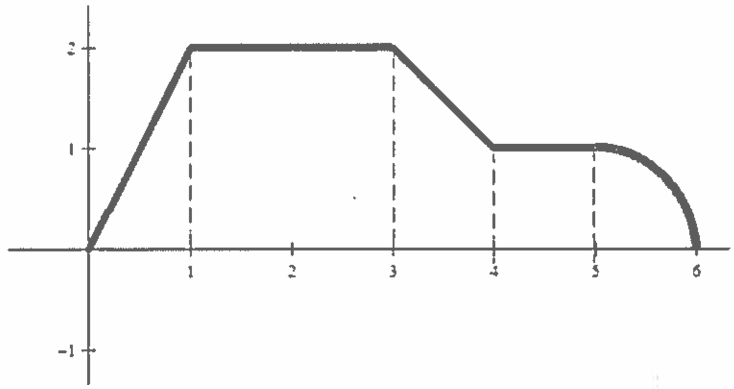
Homework: Riemann Sum worksheet

-Understand the definition of a Riemann Sum

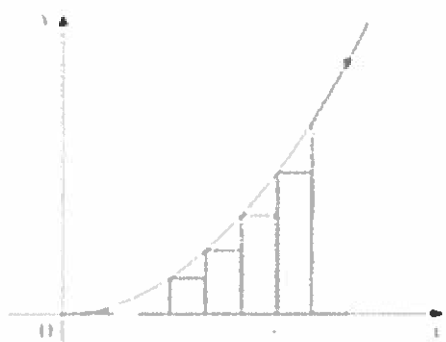
-Use Riemann Sums to find the area under a curve

How do you find the area under a graph?

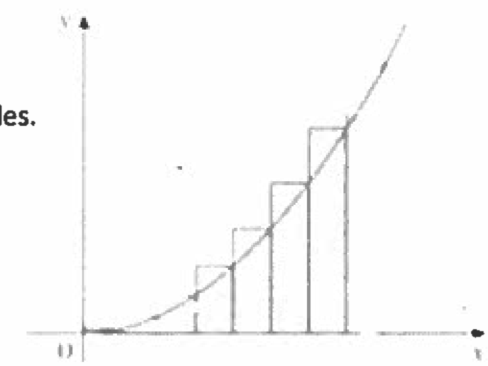
If it is a piecewise graph made from linear functions and portions of circles, then you can split the graph into geometric shapes.



What if the graph was curved?



The most common way is to use rectangles.



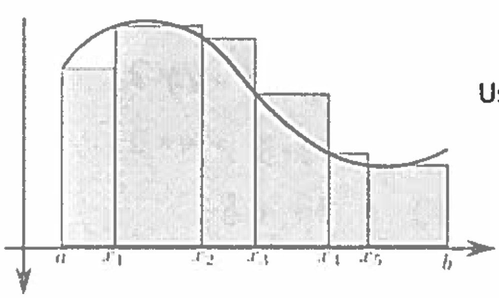
Left Riemann: the height of each rectangle comes from the left side.

Right Riemann: the height of each rectangle comes from the right side.

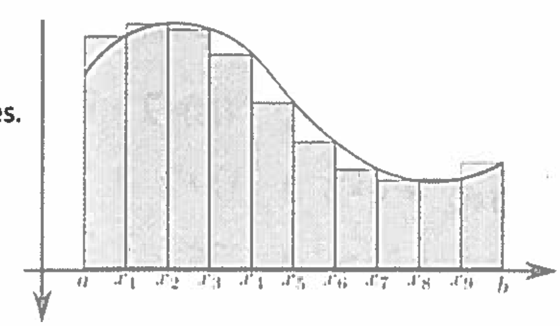
If the function is increasing, then the left Riemann sum is an underestimate and the right is an overestimate.

If the function is decreasing, then the left Riemann sum is an overestimate and the right is an underestimate.

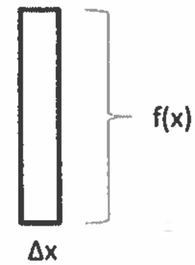
How do we get better and better estimates?



Use narrower and more rectangles.



Area of each rectangle is:



Area = f(x)Δx

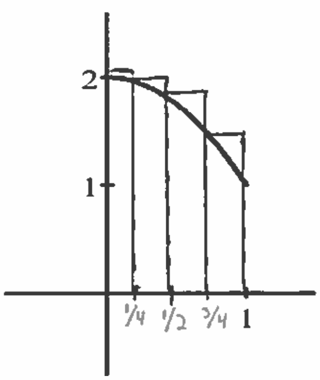
If there are n rectangles and we add them up:

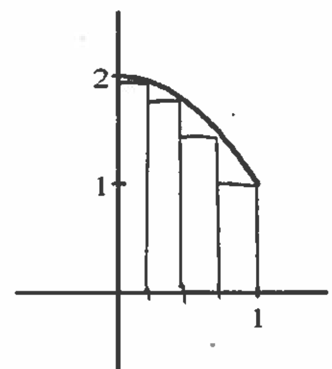
$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \ldots + f(x_{n-1})\Delta x + f(x_n)\Delta x = \sum_{i=1}^{n} f(x_i)\Delta x$$

The approximation is best if Δx is infinitely small:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$ This is the limit definition of the integral.

Ex) Use a left and right Riemann sum to approximate the area under $y = -x^2 + 2$ from (0, 1) using 4 intervals.





$A \approx f(0)\times.25 + f(1/4)\times.25 + f(1/2)\times.25 + f(3/4)\times.25$

$A \approx 2\times.25 + 1.9375\times.25 + 1.75\times.25 + 1.4375\times.25$

$\boxed{A \approx 1.78125}$

$A \approx f(1)\times.25 + f(.75)\times.25 + f(.5)\times.25 + f(.25)\times.2$

$A \approx .25(1 + 1.4375 + 1.75 + 1.9375)$

$\boxed{A \approx 1.53125}$

Use left and right Riemann sums to approximate the area under the curve f(t).

t	0	2	5	7	8
f(t)	0	4	13	21	23

Left

$A \approx f(0)\times2 + f(2)\times3 + f(5)\times2 + f(7)\times1$

$A \approx 0\times2 + 4\times3 + 13\times2 + 21\times1$

$A \approx 0 + 12 + 26 + 21$

$\boxed{A \approx 59}$

Right

$A \approx f(8)\times1 + f(7)\times2 + f(5)\times3 + f(2)\times2$

$A \approx 23\times1 + 21\times2 + 13\times3 + 4\times2$

$A \approx 23 + 42 + 39 + 8$

$\boxed{A \approx 112}$