4.4 1st Fundamental Theorem of Calculus

Page 288 #'s 5-13 odd, 25-37 odd, 103, 104, 111, 112

5) $\int_0^2 6x\,dx$

$3x^2\Big|_0^2$

$3(4) - 3(0)$

$\boxed{12}$

7) $\int_{-1}^{0} (2x-1)\,dx$

$x^2 - x\Big|_{-1}^{0}$

$(0-0) - ((-1)^2 - (-1))$

$0 - (1+1)$

$\boxed{-2}$

9) $\int_{-1}^{1} (t^2-2)\,dt$

$\frac{1}{3}t^3 - 2t\Big|_{-1}^{1}$

$\left(\frac{1}{3}(1)^2 - 2(1)\right) - \left(\frac{1}{3}(-1)^3 - 2(-1)\right)$

$\left(\frac{1}{3} - 2\right) - \left(-\frac{1}{3} + 2\right)$

$\frac{2}{3} - 4$

$\boxed{-\frac{10}{3}}$

11) $\int_0^1 (2t-1)^2\,dt$

$\int_0^1 (4t^2 - 4t + 1)\,dt$

$\frac{4}{3}t^3 - 2t^2 + t\Big|_0^1$

$\left(\frac{4}{3} - 2 + 1\right) - (0 - 0 + 0)$

$\left(\frac{1}{3}\right) - (0)$

$\boxed{1/3}$

13) $\int_1^2 (3x^{-2} - 1)\,dx$

$-3x^{-1} - x\Big|_1^2$

$\left(-\frac{3}{2} - 2\right) - \left(-\frac{3}{1} - 1\right)$

$(-3.5) - (-4)$

$\boxed{1/2}$

25) $\int_0^4 |x^2 - 9|\,dx$

$\int_0^3 -(x^2-9)\,dx + \int_3^4 (x^2-9)\,dx$

$-\frac{1}{3}x^3 + 9x\Big|_0^3 + \frac{1}{3}x^3 - 9x\Big|_3^4$

$\left(-\frac{1}{3}(27) + 9(3)\right) - (0)$

$+ \left(\frac{1}{3}(64) - 9(4)\right) - \left(\frac{1}{3}(27) - 9(3\right.$

$(-9 + 27) - (0) + \left(\frac{64}{3} - 36\right) - (9 - 2\right.$

$\boxed{\frac{64}{3}}$

27) $\int_0^{\pi} (1 + \sin x)\,dx$

$x - \cos x\Big|_0^{\pi}$

$(\pi - \cos\pi) - (0 - \cos(0))$

$(\pi - (-1)) - (-1)$

$\boxed{\pi + 2}$

29) $\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\,d\theta$

$\int_0^{\pi/4} \frac{\cos^2\theta}{\cos^2\theta}\,d\theta$

$\int_0^{\pi/4} 1\,d\theta = \theta\Big|_0^{\pi/4} = \boxed{\frac{\pi}{4}}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

31) $\int_{-\pi/6}^{\pi/6} \sec^2\theta \, d\theta$

$\tan\theta \Big|_{-\pi/6}^{\pi/6}$

$\tan\frac{\pi}{6} - \tan\left(-\frac{\pi}{6}\right)$

$\frac{1/2}{\sqrt{3}/2} - \frac{-1/2}{\sqrt{3}/2}$

$\frac{1}{\sqrt{3}/2} = \boxed{\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}}$

33) $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta \, d\theta$

$4\sec\theta \Big|_{-\pi/3}^{\pi/3}$

$\frac{4}{\cos(\pi/3)} - \frac{4}{\cos(-\pi/3)}$

$\frac{4}{1/2} - \frac{4}{1/2}$

$\boxed{0}$

35) $\int_0^1 (x - x^2)\,dx$

$\frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1$

$\left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0)$

$\boxed{\frac{1}{6}}$

37) $\int_0^{\pi/2} \cos x \, dx$

$\sin x \Big|_0^{\pi/2}$

$\sin\left(\frac{\pi}{2}\right) - \sin(0)$

$1 - 0$

$\boxed{1}$

103) $\int_0^{18} (500 - 5t)\,dt$

$500t - \frac{5}{2}t^2 \Big|_0^{18}$

$\left(500(18) - \frac{5}{2}(18)^2\right) - (0 - 0)$

$\boxed{8190 \text{ liters}}$

104) a) $\int_0^3 (4 + .75t)\,dt$

$\boxed{15.375 \text{ gallons}}$

b) $\int_3^6 (4 + .75t)\,dt$

$\boxed{22.125}$

c) Part b is larger because the oil is leaking faster as time progresses.

111) True

112) True