Calculus Section 4.4 1st Fundamental Theorem of Calculus

-Evaluate a definite integral using the Fundamental Theorem of Calculus

Homework: page 288 #'s 5-13 odd, 25-37 odd 103, 104, 111, 112

The First Fundamental Theorem of Calculus

If a function *f* is continuous on the closed interval [a, b] and *F* is an antiderivative of *f* on the interval [a, b], then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

The 1st Fundamental Theorem of Calculus is used to determine the exact area under a curve (between the curve and the x-axis).

Examples)

Evaluate each definite integral.

1) $\int_1^3 (x^2)dx$

$\frac{1}{3}x^3 \Big|_1^3$

$\frac{1}{3}(3)^3 - \frac{1}{3}(1)^3$

$\frac{1}{3}(27) - \frac{1}{3}(1)$

$9 - 1/3$

$\boxed{26/3}$

2) $\int_0^4 (\sqrt{x}+2)dx = \int_0^4 (x^{1/2}+2)dx$

$\frac{2}{3}x^{3/2} + 2x \Big|_0^4$

$\left(\frac{2}{3}(4)^{3/2} + 2(4)\right) - \left(\frac{2}{3}(0)^{3/2} + 2(0)\right)$

$\left(\frac{2}{3}(8) + 8\right) - (0)$

$\frac{16}{3} + 8$

$\boxed{40/3}$

3) $\int_0^\pi \sin x dx$

$-\cos x \Big|_0^\pi$

$-\cos(\pi) - (-\cos 0)$

$-(-1) - (-1)$

$1 + 1$

$\boxed{2}$

4) $\int_{-2}^3 (x^2 - 3x + 1)dx$

$\frac{1}{3}x^3 - \frac{3}{2}x^2 + x \Big|_{-2}^3$

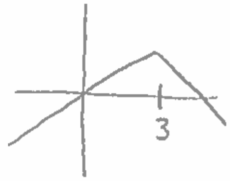
$\left(\frac{1}{3}(3)^3 - \frac{3}{2}(3)^2 + 3\right) - \left(\frac{1}{3}(-2)^3 - \frac{3}{2}(-2)^2 + (-2)\right)$

$\left(9 - \frac{27}{2} + 3\right) - \left(-\frac{8}{3} - 6 - 2\right)$

$-3/2 - (-32/3)$

$\boxed{55/6}$

5) $\int_1^4 (3 - |x-3|)dx$



$$\int_1^3 (3+(x-3))dx + \int_3^4 (3-(x-3))dx$$

$$\int_1^3 x\,dx + \int_3^4 (6-x)\,dx$$

$$\frac{1}{2}x^2\Big|_1^3 + 6x - \frac{1}{2}x^2\Big|_3^4$$

$$\left(\frac{1}{2}(3)^2 - \frac{1}{2}(1)^2\right) + \left(\left(6(4) - \frac{1}{2}(4)^2\right) - \left(6(3) - \frac{1}{2}(3)^2\right)\right)$$

$$\frac{1}{2}(9) - \frac{1}{2} + 24 - 8 - \left(18 - \frac{1}{2}(9)\right) = \boxed{6.5}$$

Example)

Determine the velocity of a particle at time t = 7 given that v(4) = 5 and a(t) = -sin(x^2 + 1)

$$\int_a^b a(t) = v(b) - v(a)$$

$$\int_4^7 -\sin(x^2+1)dx = v(7) - 5$$

$$v(7) = 5 + \int_4^7 -\sin(x^2+1)dx$$

must use calculator for integral

$$\boxed{v(7) = 5.107}$$