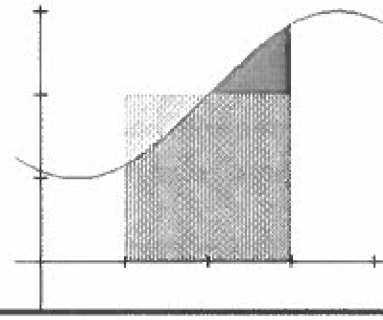
Calculus Section 4.4 Mean Value and 2nd Fund. Thm of Calculus

-Understand and use the Mean Value Theorem
-Find the average value of a function over a closed interval
-Understand and use the 2nd Fundamental Theorem of Calculus

Mean Value Thm. and 2nd FTC Worksheet

Mean Value Theorem
If *f* is continuous on the closed interval [a, b], then there exists a number *c* in the closed interval [a, b] such that

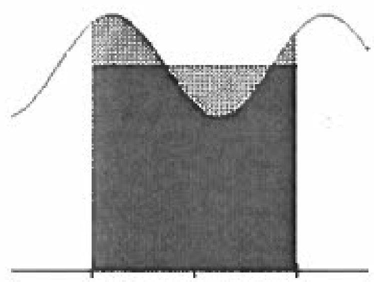
$$\int_a^b f(x)dx = f(c)(b-a)$$



In other words, there exists a *c* between [a, b] such that a rectangle of height h = f(c) would have the same area as the area found under the curve. The Mean Value Theorem only tells you f(c) exists; rearrange the equation to find the value of f(c).

Average Value of a Function
If *f* is integrable on the closed interval [a, b], then the **average value of** *f* on the interval is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$



Example)
Find the average value of $f(x) = 3x^2 - 2x$ on the interval [1, 4].

$$\frac{1}{4-1}\int_1^4 (3x^2-2x)dx$$

$$\frac{1}{3}\left[x^3 - x^2\right]_1^4$$

$$\frac{1}{3}\left[4^3 - 4^2\right] - \frac{1}{3}\left[1^3 - 1^2\right]$$

$$\frac{1}{3}(64-16) - \frac{1}{3}(0) \rightarrow \frac{1}{3}(48) \rightarrow \boxed{16}$$

The 2nd Fundamental Theorem of Calculus

If f is continuous on an open interval containing a, then, for every x in the interval: $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

If the upper limit is a function, $\frac{d}{dx}\left[\int_a^{g(x)} f(t)dt\right] = f(g(x)) \cdot g'(x)$ (chain rule)

The upper limit must be the variable. Switch the limits if x is the lower limit. $\int_x^a f(t)dt = -\int_a^x f(t)dt$

The 2nd Fundamental Theorem of Calculus ~~tells that~~ shows that if a function is continuous then it will have an antiderivative. The antiderivative may not be an elementary function.

An elementary function is written with one variable and made up of a finite number of arithmetic operations (+,-,÷,×), exponentials, logarithms, constants, and solutions of algebraic equations.

For example, $\int e^{x^2} dx$ does not have an elementary antiderivative, but it does have an antiderivative nonetheless.

Examples)

Evaluate $\frac{d}{dx}\left[\int_0^x \sqrt{t^2-1}\,dt\right]$

$\sqrt{x^2-1}$

Find the slope of $\int_{\pi/2}^{x^3} \cos(t^2)\,dt$

$\cos((x^3)^2) \cdot 3x^2$

$3x^2\cos(x^6)$

$\frac{d}{dx}\int_{\sin x}^{2} (\cos(t) + 2)dt$

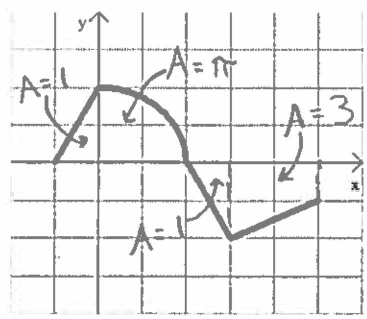
$\frac{d}{dx} -\int_2^{\sin x} (\cos t + 2)dt$

$-(\cos(\sin x) + 2)\cos x$

$-\cos(\sin x) - 2\cos x$

The graph of a function f consists of a quarter circle and line segments.

Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



(a) Find $g(0), g(-1), g(2), g(5)$.

$g(0) = \int_0^0 f(t)dt = 0$

$g(2) = \int_0^2 f(t)dt = \pi$

$g(-1) = \int_0^{-1} f(t)dt = -\int_{-1}^{0} f(t)dt = -(-1) = 1$

$g(5) = \int_0^5 f(t)dt = \pi - 1 - 3$

$g(5) = \pi - 4$

b) Find the x-coordinate of each point of inflection of the graph of g on $(-1, 5)$. Justify your answer.

$g'(x) = \frac{d}{dx}\int_0^x f(t)dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

Points of inflection will occur when the slope of $f(x)$ changes signs. $x = 0$ and $x = 3$