Mean Value Theorem and 2nd FTC Worksheet

Name: ______________________________

1. Find the derivatives of the functions defined by the following integrals:

(a) $\int_0^x \frac{\sin t}{t}\,dt$

(b) $\int_0^x e^{-t^2}\,dt$

(c) $\int_1^{\cos x} \frac{1}{t}\,dt$

(d) $\int_0^1 e^{\tan^2 t}\,dt$

(e) $\int_x^{x^2} \frac{1}{2t}\,dt, x>0$

(f) $\int_x^2 \cos\left(t^2\right)dt$

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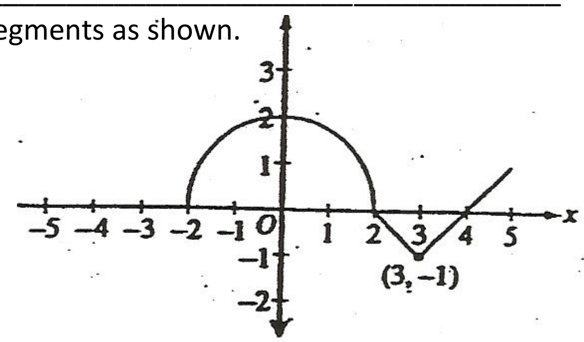
2. The graph of a function f consists of a semicircle and two line segments as shown.
Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.

(a) Find $g(0), g(3), g(-2), \text{and } g(5)$.



(b) Find all values of x on the open interval $(-2,5)$ at which g has a relative maximum. Justify your answers.

(c) Find the absolute minimum value of g on the closed interval $[-2,5]$ and the value of x at which it occurs. Justify your answer.

3. Given f(x) = $\int_{-2}^{x^2} \cos(t^2)\,dt$. Determine the second derivative of f.

4. The velocity of a car accelerating from a red light is given by v(t) = $\frac{6}{5}x^2 + \frac{1}{2}x$. Find the average velocity of the car for the first 6 seconds after the light turns green.

5. Find the average value of the functions over the given intervals.

a) $f(x) = 9 - x^2$ [-3, 3]

b) $f(x) = x^3$ [0, 1]

c) $f(x) = \sin x$ [0, π]

6. (Calculator) A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t = 0) and 8 p.m. (t = 8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(t), in hundreds of entries, at various times t are shown in the table below.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time t = 6. Show the computations leading to your answer.

(b) Use a trapezoidal sum with four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)dt$ in terms of the number of entries.

(c) At 8 p.m., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight?