

4.5 Change of Variables

Pg. 302 #'s 47-49, 57, 58, 67, 68

$$47) \int x \sqrt{x+6} dx$$

$$u = x+6 \rightarrow x = u-6 \\ du = dx$$

$$\int (u-6) u^{1/2} du$$

$$\int (u^{3/2} - 6u^{1/2}) du$$

$$\frac{2}{5} u^{5/2} - 6 \left(\frac{2}{3} u^{3/2} \right) + C$$

$$\boxed{\frac{2}{5} (x+6)^{5/2} - 4(x+6)^{3/2} + C}$$

$$48) \int x \sqrt{3x-4} dx$$

$$u = 3x-4 \quad x = \frac{u+4}{3} \\ du = 3dx \\ \frac{1}{3} du = dx$$

$$\frac{1}{3} \int \left(\frac{u+4}{3} \right) u^{1/2}$$

$$\frac{1}{9} \int u^{3/2} du + \frac{4}{9} \int u^{1/2} du$$

$$\frac{1}{9} \left(\frac{2}{5} u^{5/2} \right) + \frac{4}{9} \left(\frac{2}{3} u^{3/2} \right) + C$$

$$\boxed{\frac{2}{45} (3x-4)^{5/2} + \frac{8}{27} (3x-4)^{3/2} + C}$$

$$49) \int x^2 \sqrt{1-x} dx$$

$$u = 1-x \quad x = 1-u \\ du = -dx \quad x^2 = (1-u)^2 \\ -du = dx$$

$$-\int (1-u)^2 \sqrt{u} du$$

$$-\int (1-2u+u^2)(u^{1/2}) du$$

$$-\int (u^{1/2} - 2u^{3/2} + u^{5/2}) du$$

$$-\left(\frac{2}{3} u^{3/2} - 2 \left(\frac{2}{5} u^{5/2} \right) + \frac{2}{7} u^{7/2} \right) + C$$

$$\boxed{-\frac{2}{3} (1-x)^{3/2} + \frac{4}{5} (1-x)^{5/2} - \frac{2}{7} (1-x)^{7/2} + C}$$

$$57) \int_1^2 2x^2 \sqrt{x^3+1} dx$$

$$u = x^3+1 \\ du = 3x^2 dx$$

$$\frac{2}{3} \int_2^9 u^{1/2} du$$

$$\frac{2}{3} du = 2x^2 dx$$

$$u(2) = 9$$

$$u(1) = 2$$

$$\frac{2}{3} \left[\frac{2}{3} u^{3/2} \right]_2^9$$

$$\frac{4}{9} (9)^{3/2} - \frac{4}{9} (2)^{3/2}$$

$$\boxed{12 - \frac{4}{9} \sqrt{8}}$$

$$58) \int_0^1 x \sqrt{1-x^2} dx \quad u = 1-x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} du = x dx$$

$$u(1) = 0$$

$$u(0) = 1$$

$$-\frac{1}{2} \int_1^0 u^{1/2} du$$

$$\frac{1}{2} \int_0^1 u^{1/2} du$$

$$\frac{1}{2} \left(\frac{2}{3} u^{3/2} \right) \Big|_0^1$$

$$\frac{1}{3} (1)^{3/2} - \frac{1}{3} (0)^{3/2}$$

$$\boxed{\frac{1}{3}}$$

$$67) \int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{3}\right) dx$$

$$u = \frac{x}{3}$$

$$du = \frac{1}{3} dx$$

$$3 du = dx$$

$$u\left(\frac{2\pi}{3}\right) = \frac{\pi}{3}$$

$$u\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$$

$$2 \int_{\pi/4}^{\pi/3} \sec^2(u) du$$

$$2 \tan(u) \Big|_{\pi/4}^{\pi/3}$$

$$2 \tan\left(\frac{\pi}{3}\right) - 2 \tan\left(\frac{\pi}{4}\right)$$

$$2\sqrt{3} - 2(1)$$

$$\boxed{2\sqrt{3} - 2}$$

$$68) \int_{\pi/12}^{\pi/4} \csc 2x \cot 2x dx$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$u\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$$u\left(\frac{\pi}{12}\right) = \frac{\pi}{6}$$

$$\frac{1}{2} \int_{\pi/6}^{\pi/2} \csc(u) \cot(u) du$$

$$\frac{1}{2} \left[-\csc(u) \right]_{\pi/6}^{\pi/2}$$

$$-\frac{1}{2} \csc\left(\frac{\pi}{2}\right) - \left(-\frac{1}{2} \csc\left(\frac{\pi}{6}\right)\right)$$

$$\frac{-1}{2 \sin\left(\frac{\pi}{2}\right)} + \frac{1}{2 \sin\left(\frac{\pi}{6}\right)}$$

$$\frac{-1}{2(1)} + \frac{1}{2(1/2)} = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$$