Calculus Section 4.5 Change of Variables for Integration

-Use a change of variables to evaluate a definite integral

When you use u-substitution to evaluate a definite integral, you must also change the __limits__ of __integration__ before you can evaluate the integral.

Change of Variables for Definite Integrals

If the function u = g(x) and du = g'(x)dx, then

$$\int_a^b f(g(x))g'(x)dx = \int_{u(a)}^{u(b)} f(u)du$$

So, the new limits of integration are found by substituting a and b into the u-function.

Examples)

1) $\int_0^1 x(x^2+1)^3 dx$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$u(1) = 2$

$u(0) = 1$

$\frac{1}{2}\int_1^2 u^3 du$

$\frac{1}{2}\left[\frac{1}{4}u^4\right]_1^2$

$\frac{1}{8}(2)^4 - \frac{1}{8}(1)^4$

$2 - \frac{1}{8}$

$\boxed{\dfrac{15}{8}}$

2) $\int_1^5 \sqrt{2x-1}\,dx$

$u = 2x - 1$

$du = 2dx$

$\frac{1}{2}du = dx$

$u(5) = 9$

$u(1) = 1$

$\frac{1}{2}\int_1^9 u^{1/2} du$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^9$

$\frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2}$

$9 - \frac{1}{3}$

$\boxed{\dfrac{26}{3}}$

When there are extra variables not found in du

Examples)

1) $\int x\sqrt{2x+1}\,dx$ $u = 2x+1 \rightarrow x = \dfrac{u-1}{2}$

$\dfrac{1}{2}\int \left(\dfrac{u-1}{2}\right) u^{1/2}\,du$ $du = 2dx$

$\dfrac{1}{4}\int \left(u^{3/2} - u^{1/2}\right) du$ $\dfrac{1}{2}du = dx$

$\dfrac{1}{4}\left(\dfrac{2}{5}u^{5/2} - \dfrac{2}{3}u^{3/2} + C\right)$

$\boxed{\dfrac{1}{10}(2x+1)^{5/2} - \dfrac{1}{6}(2x+1)^{3/2} + C}$

2) $\int_0^1 \dfrac{x^3}{(x^2+3)^5}\,dx$ $u = x^2 + 3 \rightarrow x^2 = u - 3$

$du = 2x\,dx$

$\dfrac{1}{2}\int_3^4 \dfrac{u-3}{u^5}\,du$ $\dfrac{1}{2}du = x\,dx$

$\dfrac{1}{2}\int_3^4 \left(u^{-4} - 3u^{-5}\right) du$ $u(1) = 4$

$u(0) = 3$

$\dfrac{1}{2}\left[-\dfrac{1}{3}u^{-3} + \dfrac{3}{4}u^{-4}\right]_3^4$

$\dfrac{-1}{6u^3} + \dfrac{3}{8u^4}\Big|_3^4$

$\left(\dfrac{-1}{6(4)^3} + \dfrac{3}{8(4)^4}\right) - \left(\dfrac{-1}{6(3)^3} + \dfrac{3}{8(3)^4}\right)$

$\dfrac{-1}{384} + \dfrac{3}{2048} - \left(\dfrac{-1}{162} + \dfrac{3}{648}\right)$

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