4.5 Integration by substitution

Pg. 301 #'s 5-17 odd, 23, 25, 33-41 odd, 79

5) $\int (1+6x)^4 (6)\,dx$

$u = 1+6x$

$du = 6\,dx$

$\int u^4\,du$

$\frac{1}{5}u^5 + C$

$\boxed{\frac{1}{5}(1+6x)^5 + C}$

7) $\int \sqrt{25-x^2}\,(-2x)\,dx$

$u = 25-x^2$

$du = -2x\,dx$

$\int u^{1/2}\,du$

$\frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{3}(25-x^2)^{3/2} + C}$

9) $\int x^3 (x^4+3)^2\,dx$

$u = x^4+3$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int u^2\,du$

$\frac{1}{4}\left(\frac{1}{3}u^3 + C\right)$

$\boxed{\frac{1}{12}(x^4+3)^3 + C}$

11) $\int x^2 (x^3-1)^4\,dx$

$u = x^3-1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^4\,du$

$\frac{1}{3}\left(\frac{1}{5}u^5 + C\right)$

$\boxed{\frac{1}{15}(x^3-1)^5 + C}$

13) $\int t\sqrt{t^2+2}\,dt$

$u = t^2+2$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\left(\frac{2}{3}u^{3/2} + C\right)$

$\boxed{\frac{1}{3}(t^2+2)^{3/2} + C}$

15) $\int 5x\sqrt[3]{1-x^2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{5}{2}du = 5x\,dx$

$-\frac{5}{2}\int u^{1/3}\,du$

$-\frac{5}{2}\left(\frac{3}{4}u^{4/3} + C\right)$

$\boxed{\frac{-15}{8}(1-x^2)^{4/3} + C}$

17) $\int \frac{x}{(1-x^2)^3}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int u^{-3}\,du$

$-\frac{1}{2}\left(-\frac{1}{2}u^{-2} + C\right)$

$\boxed{\frac{1}{4(1-x^2)^2} + C}$

23) $\int \left(1+\frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right)dt$

$u = 1+\frac{1}{t}$

$du = -\frac{1}{t^2}\,dt$

$-du = \frac{1}{t^2}\,dt$

$-\int u^3\,du$

$-\left(\frac{1}{4}u^4 + C\right)$

$\boxed{-\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C}$

25) $\int \frac{1}{\sqrt{2x}}\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}\left(2u^{1/2} + C\right)$

$\boxed{\sqrt{2x} + C}$

33) $\int \pi \sin(\pi x)dx$

$u = \pi x$

$du = \pi dx$

$\int \sin(u)du$

$-\cos(u) + C$

$\boxed{-\cos(\pi x) + C}$

35) $\int \cos 8x\, dx$

$u = 8x$

$du = 8dx$

$\frac{1}{8}du = dx$

$\frac{1}{8}\int \cos(u)du$

$\frac{1}{8}\sin(u) + C$

$\boxed{\frac{1}{8}\sin(8x) + C}$

37) $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right)d\theta$

$u = \frac{1}{\theta}$

$du = -\frac{1}{\theta^2}d\theta$

$-du = \frac{1}{\theta^2}d\theta$

$-\int \cos(u)du$

$-\sin(u) + C$

$\boxed{-\sin\left(\frac{1}{\theta}\right) + C}$

39) $\int \sin 2x \cos 2x\, dx$

$u = \sin 2x$

$du = 2\cos 2x\, dx$

$\frac{1}{2}du = \cos 2x\, dx$

$\frac{1}{2}\int u\, du$

$\frac{1}{2}\left(\frac{1}{2}u^2 + c\right)$

$\boxed{\frac{1}{4}(\sin 2x)^2 + C}$

41) $\int \frac{\csc^2 x}{\cot^3 x}dx$

$u = \cot x$

$du = -\csc^2 x\, dx$

$-du = \csc^2 x\, dx$

$-\int u^{-3}du$

$-\left(-\frac{1}{2}u^{-2} + c\right)$

$\boxed{\frac{1}{2\cot^2 x} + C}$

79) a) $\int x^2\sqrt{x^3+1}\,dx$

b) $\int \tan 3x \sec^2 3x\, dx$

In both cases you could use u-sub to find the integral.