Calculus Section 4.5 Integration by Substitution

-Use a change of variables to find an indefinite integral

Homework: page 301 #'s 5-17 odd, 23, 25, 33-41 odd, 79

Integration by substitution is the way to integrate a function whose derivative involved the chain rule.

Antidifferentiation of a Composite Function (U-Substitution)

$$\int f(g(x))g'(x)dx = F(g(x)) + C$$

Or, if you allow u = g(x), then du = g'(x)dx and

$$\int f(u)du = F(u) + C$$

The key to solving any u-substitution problem is to look for a u-function whose ____________________ will match up with other parts of the function. We're trying to simplify each integrand into something that we've seen before and already know how to take the integral of using the integral "power rule."

Examples)

1) $\int 2x(x^2+1)^4 dx$

$\int (x^2+1)^4 \, 2x\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\int u^4 du$

$\frac{1}{5}u^5 + C$

$\boxed{\frac{1}{5}(x^2+1)^5 + C}$

2) $\int 3x^2\sqrt{x^3+1}dx$

$\int (x^3+1)^{1/2} \, 3x^2 dx$

$u = x^3+1$

$du = 3x^2 dx$

$\int u^{1/2} du$

$\frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{3}(x^3+1)^{3/2} + C}$

3) $\int \sec^2 x(\tan x$ [redacted] $)dx$

$\int u\,du$

$u = \tan x$

$du = \sec^2 x\,dx$

$\frac{1}{2}u^2 + C$

$\boxed{\frac{1}{2}\tan^2 x + C}$

4) $\int 5\cos(5x)dx$

$\int \cos(u)du$

$u = 5x$

$du = 5dx$

$\sin(u) + C$

$\boxed{\sin(5x) + C}$

More Examples)

5) $\int x(x^2+1)^2\,dx$

$\frac{1}{2}\int u^2\,du$

$\frac{1}{2}\left(\frac{1}{3}u^3+c\right)$

$\boxed{\frac{1}{6}(x^2+1)^3+C}$

$u=x^2+1$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

6) $\int 2x^2\sqrt{x^3+2}\,dx$

$\frac{2}{3}\int u^{1/2}\,du$

$\frac{2}{3}\left(\frac{2}{3}u^{3/2}+C\right)$

$\boxed{\frac{4}{9}(x^3+2)^{3/2}+c}$

$u=x^3+2$

$du=3x^2\,dx$

$\frac{1}{3}du=x^2\,dx$

$\frac{2}{3}du=2x^2\,dx$

7) $\int 2\sec^2 x(\tan x+3)\,dx$

$2\int u\,du$

$2\left(\frac{1}{2}u^2+c\right)$

$\boxed{(\tan x+3)^2+C}$

$u=\tan x+3$

$du=\sec^2 x\,dx$

$2du=2\sec^2 x\,dx$