

Calculus Section 5.2 Trig Integration

Integrate trigonometric functions

Homework: page 334 #'s 31-40, 49, 55, 100

1) $\int \sin x dx$

$$-\cos x + C$$

2) $\int \cos x dx$

$$\sin x + C$$

3) $\int \tan x dx$

$$\int \frac{\sin x}{\cos x} dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$-\int \frac{1}{u} du$$

$$-\ln|u| + C$$

$$-\ln|\cos x| + C$$

4) $\int \cot x dx$

$$\int \frac{\cos x}{\sin x} dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$\int \frac{1}{u} du$$

$$\ln|u| + C$$

$$\ln|\sin x| + C$$

5) $\int \sec x dx$

$$\int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) dx$$

$$\int \frac{1}{u} du$$

$$\ln|\sec x + \tan x| + C$$

6) $\int \csc x dx$

$$\int \csc x \left(\frac{\csc x + \cot x}{\csc x + \cot x} \right) dx$$

$$\int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} dx$$

$$u = \csc x + \cot x$$

$$du = (-\csc x \cot x - \csc^2 x) dx$$

$$-du = (\csc x \cot x + \csc^2 x) dx$$

$$-\int \frac{1}{u} du$$

$$-\ln|\csc x + \cot x| + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \csc^2 u du = -\cot u + C$$

Example)

$$1) \int \frac{\sec^2 x}{\tan x} dx$$

$$u = \tan x \\ du = \sec^2 x dx$$

$$\int \frac{1}{u} du$$

$$\ln|u| + C$$

$$\boxed{\ln|\tan x| + C}$$

Pythagorean Identity
 $1 + \tan^2 x = \sec^2 x$

$$2) \int \sqrt{1 + \tan^2 x} dx$$

$$\int \sqrt{\sec^2 x} dx$$

$$\int \sec x dx$$

$$\boxed{\ln|\sec x + \tan x| + C}$$

$$3) \int \csc(4x) dx$$

$$u = 4x$$

$$du = 4dx$$

$$\frac{1}{4} \int \csc u du$$

$$\frac{1}{4} du = dx$$

$$-\frac{1}{4} \ln|\csc u + \cot u| + C$$

$$\boxed{-\frac{1}{4} \ln|\csc 4x + \cot 4x| + C}$$

4) Find the average value of $f(x) = \tan(x)$ on the interval $[0, \pi/4]$

$$\frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \tan x dx$$

$$-\frac{1}{\frac{\pi}{4}} \ln|\cos x| \Big|_0^{\pi/4}$$

$$-\frac{4}{\pi} \ln|\cos \frac{\pi}{4}| - \left(-\frac{4}{\pi} \ln|\cos 0|\right)$$

$$-\frac{4}{\pi} \ln\left(\frac{\sqrt{2}}{2}\right) + \frac{4}{\pi} \ln(1)$$

$$\boxed{-\frac{4}{\pi} \ln\left(\frac{\sqrt{2}}{2}\right)}$$