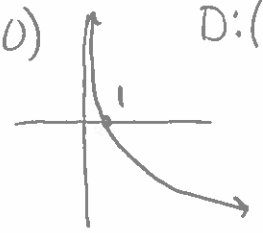
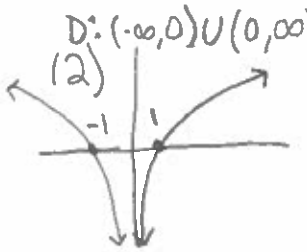
5.1 The Natural Log

Pg. 325 #'s 10, 12-14, 19, 21, 23, 29-31, 41-49 odd, 61, 62, 67, 83, 99-102

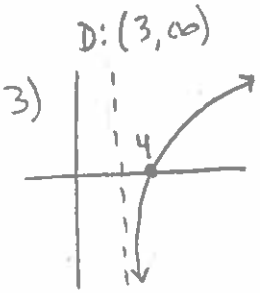
10) D: $(0,\infty)$



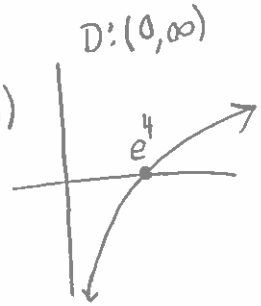
(12) D: $(-\infty,0)\cup(0,\infty)$



13) D: $(3,\infty)$



14) D: $(0,\infty)$



19) $\ln\left(\frac{x}{4}\right)$

$\ln x - \ln 4$

21) $\ln\frac{xy}{z}$

$\ln x + \ln y - \ln z$

23) $\ln\left(x\cdot(x^2+5)^{1/2}\right)$

$\ln x + \frac{1}{2}\ln(x^2+5)$

29) $\ln(x-2) - \ln(x+2)$

$\ln\left(\frac{x-2}{x+2}\right)$

30) $3\ln x + 2\ln y - 4\ln z$

$\ln\left(\frac{x^3y^2}{z^4}\right)$

31) $\frac{1}{3}\left[2\ln(x+3) + \ln x - \ln(x^2-1)\right]$

$\ln\sqrt[3]{\frac{(x+3)^2 x}{x^2-1}}$

41) $f(x) = \ln(3x)$

$f'(x) = \frac{1}{3x}(3)$

$f'(x) = \frac{1}{x}$

43) $f(x) = 2\ln x$

$f'(x) = 2\left(\frac{1}{x}\right)(1)$

$f'(x) = \frac{2}{x}$

45) $y = (\ln x)^4$

$y' = 4(\ln x)^3 \cdot \frac{1}{x}$

$y' = \frac{4(\ln x)^3}{x}$

47) $y = \ln(t+1)^2$

$y = 2\ln(t+1)$

$y' = 2\left(\frac{1}{t+1}\right)(1)$

$y' = \frac{2}{t+1}$

49) $y = \ln\left(x\sqrt{x^2-1}\right)$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{2}\left(\frac{1}{x^2-1}\right)(2x)$

$y' = \frac{1}{x} + \frac{x}{x^2-1}$

61) $y = \ln|\sin x|$

$y' = \frac{1}{\sin x}(\cos x)$

$y' = \frac{\cos x}{\sin x}$

$y' = \cot x$

62) $y = \ln|\csc x|$

$y' = \frac{1}{\csc x}(-\csc x \cot x)$

$y' = -\cot x$

67) $f(x) = 3x^2 - \ln x$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 6(1) - \frac{1}{(1)}$

$f'(1) = 6 - 1 = 5$

$y - 3 = 5(x-1)$

83) $y = \frac{x}{\ln x}$

$y' = \frac{\ln x(1) - x(\frac{1}{x})}{(\ln x)^2}$

$y' = \frac{\ln x - 1}{(\ln x)^2}$

$\frac{\ln x - 1}{(\ln x)^2} = 0$

$\ln x - 1 = 0$

$\ln x = 1$

$x = e$

x	1/2	1	2	e	5
y	−	undef.	−	0	+

relative minimum at $x = e$

$y'' = \frac{(\ln x)^2(\frac{1}{x}) - (\ln x - 1)(2\ln x)(\frac{1}{x})}{(\ln x)^4}$

$y'' = \frac{\frac{1}{x}(\ln x)^2 - \frac{2}{x}(\ln x)^2 + \frac{2}{x}\ln x}{(\ln x)^4}$

$\frac{1}{x}(\ln x)^2 - \frac{2}{x}(\ln x)^2 + \frac{2}{x}\ln x = 0$

$-\frac{1}{x}(\ln x)^2 + \frac{2}{x}\ln x = 0$

$-(\ln x)^2 + 2\ln x = 0$

$-\ln x(\ln x - 2) = 0$

$-\ln x = 0$ $\quad$ $\ln x - 2 = 0$

$x = 1$ $\quad$ $\ln x = 2$

$x = e^2$

x	1/2	1	2	e^2	10
y''	−	0	+	0	−

Point of inflection at $x = e^2$

($x = 1$ DNE)

99) False, cannot split addition in logs

100) False, $\ln(xy) = \ln x + \ln y$

101) False, $\ln\pi$ is a constant, so $y' = 0$

102) False, $\ln(e) = 1$ and $y' = 0$