Calculus Section 5.1 The Natural Log

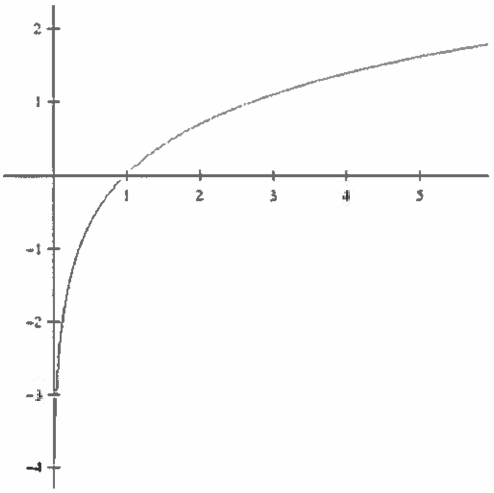
-Develop and use properties of the natural logarithmic function
-Find derivatives of functions involving the natural log

Homework: page 325 #'s 10, 12-14, 19, 21, 23, 29-31, 41-49 odd, 61, 62, 67, 83, 99-102

Definition of the Natural Logarithmic Function
The natural logarithmic function is defined by:

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

The domain of the natural logarithmic function is the set of all positive real numbers.



Properties of the ln function

$\ln(1)=0$ $\quad$ $\ln(e)=1$

$\lim_{x\to 0^+} \ln x = -\infty$ $\quad$ $\lim_{x\to\infty} \ln x = \infty$

$\ln x$ is always increasing

$\ln x$ is always concave down

Logarithmic Properties
If *a* and *b* are positive numbers and *n* is rational, then the following properties are true.

1) $\ln(1) = 0$
2) $\ln(ab) = \ln a + \ln b$
3) $\ln(a^n) = n\ln(a)$
4) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Derivative of the Natural Log Function
Let *u* be a differentiable function of x.

chain rule

1) $\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad$ 2) $\frac{d}{dx}[\ln u] = \frac{1}{u}du$ $\quad$ 3) $\frac{d}{dx}[\ln|u|] = \frac{1}{u}du$

Examples)

1) Find the eq. of the tangent line to $y = \ln(2x)$ when x = 2.

$y' = \frac{1}{2x}(2)$ $y(2) = \ln(4)$

$y' = \frac{1}{x}$ $y'(2) = \frac{1}{2}$

$\boxed{y - \ln 4 = \frac{1}{2}(x-2)}$

2) $\frac{d}{dx}[\ln 5]$ constant

$\boxed{\text{zero}}$

3) Find $f'(x)$ for $f(x) = \ln\sqrt{x+1}$

$f(x) = \ln(x+1)^{1/2}$

$f(x) = \frac{1}{2}\ln(x+1)$

$f'(x) = \frac{1}{2}\left(\frac{1}{x+1}\right)(1)$

$\boxed{f'(x) = \frac{1}{2x+2}}$

4) Find $\frac{dy}{dx}$ for $\ln(y) = x^2 \ln x$

$\frac{1}{y}\frac{dy}{dx} = x^2\left(\frac{1}{x}\right) + \ln x\,(2x)$

$\frac{1}{y}\frac{dy}{dx} = x + 2x\ln x$

$\boxed{\frac{dy}{dx} = (x + 2x\ln x)y}$

5) Find the 2nd derivative of $\ln(\ln(x))$

$y = \ln(\ln(x))$

$y' = \frac{1}{\ln x}\left(\frac{1}{x}\right)$

$y' = \frac{1}{x\ln x}$

$y'' = \frac{x\ln x(0) - 1(x(\frac{1}{x}) + \ln x(1))}{(x\ln x)^2}$

$\boxed{y'' = \frac{-1-\ln x}{(x\ln x)^2}}$

6) Find $\frac{dy}{dx}$ if $y = \frac{x(x^2+1)^2}{\sqrt{2x^3-1}}$

$\ln y = \ln\left(\frac{x(x^2+1)^2}{(2x^3-1)^{1/2}}\right)$

$\ln y = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + 2\left(\frac{1}{x^2+1}\right)(2x) - \frac{1}{2}\left(\frac{1}{2x^3-1}\right)(6x^2)$

$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}\right)y$

$\boxed{\frac{dy}{dx} = \left(\frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}\right)\left(\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}\right)}$