Calculus Section 5.2 Natural Log Integration

-Find integrals of functions involving the natural log

Log Rule for Integration

Let u be a differentiable function of x.

1) $\int \frac{1}{x} dx = \ln|x| + C$

2) $\int \frac{1}{u} du = \ln|u| + C$

Examples)

1) $\int \frac{2}{x} dx$

$2 \int \frac{1}{x} dx$

$\boxed{2 \ln|x| + C}$

2) $\int \frac{1}{4x-1} dx$

$u = 4x - 1$

$du = 4 dx$

$\frac{1}{4} du = dx$

$\frac{1}{4} \int \frac{1}{u} du$

$\frac{1}{4} \ln|u| + C$

$\boxed{\frac{1}{4} \ln|4x-1| + C}$

3) $\int \frac{2x}{(x+1)^2} dx$ $\quad u = x+1 \rightarrow x = u-1$

$\qquad\qquad\qquad\qquad du = dx$

$\int \frac{2(u-1)}{u^2} du$

$\int \frac{2u-2}{u^2} du$

$\int \frac{2u}{u^2} du - \int \frac{2}{u^2} du$

$2 \int \frac{1}{u} du - 2 \int u^{-2} du$

$2 \ln|u| + 2u^{-1} + C$

$\boxed{2 \ln|x+1| + \frac{2}{x+1} + C}$

4) $\int_2^e \frac{1}{x \ln x} dx$ $\quad u = \ln x \qquad u(e) = 1$

$\qquad\qquad\qquad\qquad du = \frac{1}{x} dx \qquad u(2) = \ln 2$

$\int_{\ln 2}^{1} \frac{1}{u} du$

$\ln|u| \Big|_{\ln 2}^{1}$

$\ln|1| - \ln(\ln 2)$

$\boxed{-\ln(\ln 2)}$