5.2 Trig Integration

Pg. 334 #'s 31-40, 49, 55, 100

31) $\int \cot\frac{\theta}{3}\,d\theta$ $\quad u=\frac{\theta}{3}$, $du=\frac{1}{3}d\theta$, $3du=d\theta$

$3\int \cot u\,du$

$3\ln|\sin u|+C$

$\boxed{3\ln\left|\sin\frac{\theta}{3}\right|+C}$

32) $\int \tan 5\theta\,d\theta$ $\quad u=5\theta$, $du=5d\theta$, $\frac{1}{5}du=d\theta$

$\frac{1}{5}\int \tan u\,du$

$-\frac{1}{5}\ln|\cos u|+C$

$\boxed{-\frac{1}{5}\ln|\cos 5\theta|+C}$

33) $\int \csc 2x\,dx$ $\quad u=2x$, $du=2dx$, $\frac{1}{2}du=dx$

$\frac{1}{2}\int \csc u\,du$

$-\frac{1}{2}\ln|\csc u+\cot u|+C$

$\boxed{-\frac{1}{2}\ln|\csc 2x+\cot 2x|+C}$

34) $\int \sec\frac{x}{2}\,dx$ $\quad u=\frac{x}{2}$, $du=\frac{1}{2}dx$, $2du=dx$

$2\int \sec u\,du$

$2\ln|\sec u+\tan u|+C$

$\boxed{2\ln\left|\sec\frac{x}{2}+\tan\frac{x}{2}\right|+C}$

35) $\int(\cos 3\theta-1)\,d\theta$ $\quad u=3\theta$, $du=3d\theta$, $\frac{1}{3}du=d\theta$

$\frac{1}{3}\int \cos u\,du-\int d\theta$

$\frac{1}{3}\sin u-\theta+C$

$\boxed{\frac{1}{3}\sin 3\theta-\theta+C}$

36) $\int\left(2-\tan\frac{\theta}{4}\right)d\theta$ $\quad u=\frac{\theta}{4}$, $du=\frac{1}{4}d\theta$, $4du=d\theta$

$\int 2d\theta-4\int \tan u\,du$

$2\theta+4\ln|\cos u|+C$

$\boxed{2\theta+4\ln\left|\cos\frac{\theta}{4}\right|+C}$

37) $\int\frac{\cos t}{1+\sin t}\,dt$ $\quad u=1+\sin t$, $du=\cos t\,dt$

$\int\frac{1}{u}\,du$

$\ln|u|+C$

$\boxed{\ln|1+\sin t|+C}$

38) $\int\frac{\csc^2 t}{\cot t}\,dt$ $\quad u=\cot t$, $du=-\csc^2 t$, $-du=\csc^2 t$

$-\int\frac{1}{u}\,du$

$-\ln|u|+C$

$\boxed{-\ln|\cot(t)|+C}$

39) $\int\frac{\sec x\tan x}{\sec x-1}\,dx$ $\quad u=\sec x-1$, $du=\sec x\tan x\,dx$

$\int\frac{1}{u}\,du$

$\ln|u|+C$

$\boxed{\ln|\sec x-1|+C}$

40) $\int(\sec 2x + \tan 2x)\,dx$ $\quad u = 2x$, $du = 2dx$, $\frac{1}{2}du = dx$

$\frac{1}{2}\int(\sec u + \tan u)\,du$

$\frac{1}{2}\ln|\sec u + \tan u| - \frac{1}{2}\ln|\cos u| + C$

$\boxed{\frac{1}{2}\ln|\sec 2x + \tan 2x| - \frac{1}{2}\ln|\cos 2x| + C}$

49) $\int_0^4 \frac{5}{3x+1}\,dx$ $\quad u = 3x+1$, $du = 3dx$, $\frac{1}{3}du = dx$, $u(4) = 13$, $u(0) = 1$

$\frac{5}{3}\int_1^{13} \frac{1}{u}\,du$

$\frac{5}{3}\ln|u|\Big|_1^{13}$

$\frac{5}{3}\ln(13) - \frac{5}{3}\ln(1)$

$\boxed{\frac{5}{3}\ln(13)}$

55) $\int_1^2 \frac{1-\cos\theta}{\theta - \sin\theta}\,d\theta$ $\quad u = \theta - \sin\theta$, $du = (1-\cos\theta)\,d\theta$, $u(2) = 2 - \sin 2$, $u(1) = 1 - \sin 1$

$\int_{1-\sin 1}^{2-\sin 2} \frac{1}{u}\,du$

$\ln|u|\Big|_{1-\sin 1}^{2-\sin 2}$

$\boxed{\ln|2-\sin 2| - \ln|1-\sin 1|}$

100) $P = \frac{90000}{400+3x}$ $\quad u = 400 + 3x$, $du = 3dx$, $\frac{1}{3}du = dx$, $u(50) = 550$, $u(40) = 520$

$\frac{1}{50-40}\cdot\frac{90000}{3}\int_{520}^{550} \frac{1}{u}\,du$

$\frac{1}{10}\cdot 30000\ln|u|\Big|_{520}^{550}$

$\boxed{3000\ln|550| - 3000\ln|520|}$

$\approx \$168.27$