

5.3 Derivative of Inverse Functions

Pg. 344 #'s 63-68

63) $f(x) = 5 - 2x^3$ $(f^{-1})'(7) = ?$

$f'(x) = -6x^2$

$f'(-1) = -6(-1)^2 = -6$

$(f^{-1})'(7) = \frac{1}{f'(-1)} = \frac{1}{-6} = -\frac{1}{6}$

$\frac{f(x)}$	$\frac{f^{-1}(x)}$
$(?, 7)$	$(7, ?)$
$(-1, 7)$	$(7, -1)$

$7 = 5 - 2x^3$
 $2 = -2x^3$
 $-1 = x^3$
 $-1 = x$

$(f^{-1})'(7) = -\frac{1}{6}$

64) $f(x) = x^3 + 2x - 1$ $(f^{-1})'(2) = ?$

$f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2 = 5$

$(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{5}$

$\frac{f(x)}$	$\frac{f^{-1}(x)}$
$(?, 2)$	$(2, ?)$
$(1, 2)$	$(2, 1)$

$2 = x^3 + 2x - 1$
 $3 = x^3 + 2x$
 $x = 1$

$(f^{-1})'(2) = \frac{1}{5}$

65) $f(x) = \frac{1}{27}x^5 + \frac{2}{27}x^3$ $(f^{-1})'(-11) = ?$

$f'(x) = \frac{5}{27}x^4 + \frac{6}{27}x^2$

$f'(-3) = \frac{5}{27}(-3)^4 + \frac{6}{27}(-3)^2$

$f'(-3) = 17$

$(f^{-1})'(-11) = \frac{1}{f'(-3)} = \frac{1}{17}$

$\frac{f(x)}$	$\frac{f^{-1}(x)}$
$(?, -11)$	$(-11, ?)$
$(-3, -11)$	$(-11, -3)$

$-11 = \frac{1}{27}(x^5 + 2x^3)$
 $-297 = x^5 + 2x^3$
 $x = -3$

$(f^{-1})'(-11) = \frac{1}{17}$

$$66) f(x) = \sqrt{x-4} \quad (f^{-1})'(2) = ?$$

$$f(x) = (x-4)^{1/2}$$

$$f'(x) = \frac{1}{2}(x-4)^{-1/2}$$

$$f'(8) = \frac{1}{2\sqrt{8-4}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$(f^{-1})'(2) = \frac{1}{f'(8)} = \frac{1}{1/4}$$

$$\frac{f(x)}{f^{-1}(x)}$$

$$(\cdot, 2) \quad (2, \cdot)$$

$$(8, 2) \quad (2, 8)$$

$$2 = \sqrt{x-4}$$

$$4 = x-4$$

$$x = 8$$

$$(f^{-1})'(2) = 4$$

$$67) f(x) = \sin x \quad (f^{-1})'(1/2) = ?$$

$$f'(x) = \cos x$$

$$f'(\pi/6) = \frac{\sqrt{3}}{2}$$

$$(f^{-1})'(1/2) = \frac{1}{f'(\pi/6)} = \frac{1}{\sqrt{3}/2}$$

$$\frac{f(x)}{f^{-1}(x)}$$

$$(\cdot, 1/2) \quad (1/2, \cdot)$$

$$(\pi/6, 1/2) \quad (1/2, \pi/6)$$

$$1/2 = \sin x$$

$$x = \pi/6$$

$$(f^{-1})'(1/2) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$68) f(x) = \cos 2x \quad (f^{-1})'(1) = ?$$

$$f'(x) = -2\sin 2x$$

$$f'(0) = -2\sin(0) = 0$$

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{0}$$

$$\frac{f(x)}{f^{-1}(x)}$$

$$(\cdot, 1) \quad (1, \cdot)$$

$$(0, 1) \quad (1, 0)$$

$$1 = \cos 2x$$

$$x = 0$$

$(f^{-1})'(1)$ is undefined