Calculus Section 5.3 Derivative of Inverse Functions

-Determine the derivative of an inverse function

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The Derivative of an Inverse Function

Let f be a function that is differentiable on an interval. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$, and $g'(x) = \dfrac{1}{f'(g(x))}$.

Proof:

$\frac{d}{dx}\left[F(g(x)) = x\right]$

$f'(g(x))g'(x) = 1$

$g'(x) = \dfrac{1}{f'(g(x))}$

The derivative of $f^{-1}(x)$ at (a,b) is the reciprocal of the derivative of $f(x)$ at (b,a).

Example

Let f(x) = 2x + 5. Find $(f^{-1})'(9)$.

$f'(x) = 2$

$f'(2) = 2$

$f(x)$	$f^{-1}(x)$
$(?,9)$	$(9,?)$
$(2,9)$	$(9,2)$

$9 = 2x+5$

$4 = 2x$

$2 = x$

$(f^{-1})'(9) = \dfrac{1}{f'(2)}$

$\boxed{(f^{-1})'(9) = \frac{1}{2}}$

Example)

$(f^{-1})'(3)?$

Let $f(x)=\frac{1}{4}x^3+x-1$. What is the value of ~~$(f^{-1})'(x)$ when $x=3$~~?

$f(x)$	$f^{-1}(x)$
$(?,3)$	$(3,?)$
$(2,3)$	$(3,2)$

$3=\frac{1}{4}x^3+x-1$

$4=\frac{1}{4}x^3+x$

$x=2$

$f'(x)=\frac{3}{4}x^2+1$

$f'(2)=\frac{3}{4}(2)^2+1$

$f'(2)=4$

$(f^{-1})'(3)=\frac{1}{f'(2)}$

$\boxed{(f^{-1})'(3)=\frac{1}{4}}$

Example)

Values of f(x), f'(x), and $f^{-1}(x)$ are given in the table below. Determine $(f^{-1})'(6)$.

x	2	3	4	6
f(x)	6	11	18	38
f'(x)	4	6	8	12
$f^{-1}(x)$	0	1	1.4	2

$f(x)$	$f^{-1}(x)$
$(?,6)$	$(6,?)$
$(2,6)$	$(6,2)$

$f'(2)=4$

$(f^{-1})'(6)=\frac{1}{f'(2)}$

$\boxed{(f^{-1})'(6)=\frac{1}{4}}$