Calculus Section 5.3 Inverse Functions

-Verify that one function is the inverse function of another
-Determine whether a function has an inverse

Homework: page 343 #'s 1, 5, 7, 35-41 odd (only part a), 47, 87, 89-92

Definition of the Inverse of a Function
A function *g* is the **inverse function** of the function *f* if $f(g(x))=x$ and $g(f(x))=x$.

If *g* is the inverse of *f*, then *f* is the inverse of g.

The function g is denoted by $f^{-1}(x)$.

A function does not have to have an inverse function, but if it does, the inverse function is unique.

If a function has an inverse, then the inverse can be found by switching the x and y variables and solving for y.

Example)
Find the inverse of $f(x) = 2x^3 - 1$ and verify they are inverses using composition.

$y = 2x^3 - 1$

$x = 2y^3 - 1$

$x+1 = 2y^3$

$\frac{x+1}{2} = y^3$

$\sqrt[3]{\frac{x+1}{2}} = y$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$

$f(f^{-1}(x)) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$

$f(f^{-1}(x)) = 2\left(\frac{x+1}{2}\right) - 1$

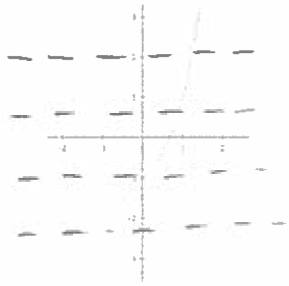
$f(f^{-1}(x)) = x + 1 - 1$

$f(f^{-1}(x)) = x$

Existence of an Inverse Function: The Horizontal Line Test
If any horizontal line crosses a function more than once, then the function fails the horizontal line test and does not have an inverse function.

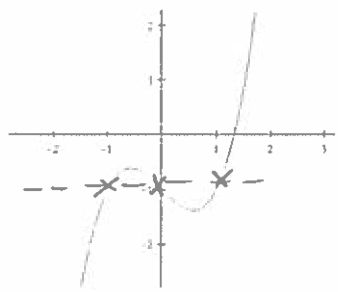
$x^3 + x - 1$

has an inverse



$x^3 - x - 1$

does not have an inverse



Properties on a Function and Its Inverse

1) If f is continuous on its domain, then f^{-1} is continuous on its domain.

2) If f is increasing on its domain, then f^{-1} is increasing on its domain.

3) If f is decreasing on its domain, then f^{-1} is decreasing on its domain.

4) If f is differentiable on an interval containing c and $f'(c) \neq 0$, then f^{-1} is differentiable at f(c).

5) If f contains the point (a,b), then f^{-1} contains the point (b,a).

6) The graphs of f and f^{1} are reflections over the line y=x.